U.S. DEPARTMENT OF COMMERCE
National Technical Information Service

PB-266 255

An Objective Analysis Technique for the Regional Air Pollution Study. Part I

Control Data Corp, Minneapolis, Minn

Prepared for

Environmental Sciences Research Lab, Research Triangle Park, N C
Meteorology and Assessment Div

Jan 77

PB 266 255

EPA-600/4-77-002a
January 1977

Environmental Monitoring Series

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U. S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

RESEARCH REPORTING SERIES

Research reports of the Office of Research and Development, U.S. Environmental Protection Agency, have been grouped into five series. These five broad categories were established to facilitate further development and application of environmental technology. Elimination of traditional grouping was consciously planned to foster technology transfer and a maximum interface in related fields. The five series are:

1. Environmental Health Effects Research
2. Environmental Protection Technology
3. Ecological Research
4. Environmental Monitoring
5. Socioeconomic Environmental Studies

This report has been assigned to the ENVIRONMENTAL PROTECTION TECHNOLOGY series. This series describes research performed to develop and demonstrate instrumentation, equipment, and methodology to repair or prevent environmental degradation from point and non-point sources of pollution. This work provides the new or improved technology required for the control and treatment of pollution sources to meet environmental quality standards.

This document is available to the public through the National Technical Information Service, Springfield, Virginia 22161.

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO.	2.	3. RECIPIENT'S ACCESSION NO.
EPA-600/4-77-002a		

4. TITLE AND SUBTITLE	5. REPORT DATE
AN OBJECTIVE ANALYSIS TECHNIQUE FOR THE REGIONAL AIR POLLUTION STUDY Part I	January 1977 6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) D. Hovland, D. Dartt, and K. Gage	8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Control Data Corporation 8100 South 34th Avenue Minneapolis, MN 5440	10. PROGRAM ELEMENT NO. 1AA603 11. CONTRACT/GRANT NO. 68-02-1827
12. SPONSORING AGENCY NAME AND ADDRESS Environmental Sciences Research Laboratories Office of Research & Development U.S. Environmental Protection Agency Research Triangle Park, N.C. 27711	13. TYPE OF REPORT AND PERIOD COVERED Final 14. SPONSORING AGENCY CODE EPA/600/09

15. SUPPLEMENTARY NOTES
Part II of this report has been issued as EPA-600/4-77-002b, February 1977.

16. ABSTRACT

This report documents the development of an objective analysis program for the mesoscale gridding of wind and temperature for the Regional Air Pollution Study being conducted in St. Louis by the Environmental Protection Agency. The program is designed to produce a 5-km spaced horizontal grid analysis from a distribution of observations which are sparse at the boundaries of the grid and dense near the center. An iterative scan procedure is used successively to correct an initial guess field until the analysis agrees reasonably well with observations. A procedure is used where widely spaced observations and a large scan radius are first used to approximate the field. This is successively followed by the addition of more observational data and reduction in scan radius until the field converges to the desired analysis (usually five iterations are required). This procedure of simultaneously adding more data and shrinking the scan radius insures that the small-scale variability in areas of dense observations does not propagate into the surrounding areas where there are few data.

The special problems of producing three-dimensional fields of gridded data from the observation network are discussed. They include the inconsistency of the surface and upper air observation networks, the non-uniform density of the basic observing network, and the difficulty of producing a reliable analysis when data from one or more key stations are missing.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
*Air pollution *Meteorological data *Wind (meteorology) *Temperature *Grids (coordinates) *Atmospheric models		13B 04B 08B 04A

18. DISTRIBUTION STATEMENT	19. SECURITY CLASS *(This Report)*	21. NO. OF PAGES
RELEASE TO PUBLIC	UNCLASSIFIED	55
	20. SECURITY CLASS *(This page)* UNCLASSIFIED	22. PRICE PC A04 - MF A01

EPA Form 2220-1 (9-73)

EPA-600/4-77-002a
January 1977

AN OBJECTIVE ANALYSIS TECHNIQUE FOR THE REGIONAL AIR POLLUTION STUDY PART I

by

D. Hovland
D. Dartt
K. Gage

Control Data Corporation
Minneapolis, MN 55440

68-02-1827

Project Officer

R. E. Eskridge
Meteorology and Assessment Division
Environmental Sciences Research Laboratory
Research Triangle Park, NC 27711

ENVIRONMENTAL SCIENCES RESEARCH LABORATORY
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
RESEARCH TRIANGLE PARK, NC 27711

ia

DISCLAIMER

This report has been reviewed by the Environmental Science Research Laboratory, U. S. Environmental Protection Agency, and approved for publication. Approval does not signify that the contents necessarily reflect the views and policies of the U. S. Environmental Protection Agency, nor does mention of trade names or commercial products constitute endorsement or recommendation for use.

ABSTRACT

This report documents the development of an objective analysis program for the mesoscale gridding of wind and temperature for the Regional Air Pollution Study being conducted in St. Louis by the Environmental Protection Agency. The program is designed to produce a 5-km spaced horizontal grid analysis from a distribution of observations which are sparse at the boundaries of the grid and dense near the center. An iterative scan procedure is used successively to correct an initial guess field until the analysis agrees reasonably well with observations. A procedure is used where widely spaced observations and a large scan radius are first used to approximate the field. This is successively followed by the addition of more observational data and reduction in scan radius until the field converges to the desired analysis (usually five iterations are required). This procedure of simultaneously adding more data and shrinking the scan radius insures that the small-scale variability in areas of dense observations does not propagate into the surrounding areas where there are few data.

The special problems of producing three-dimensional fields of gridded data from the observation network are discussed. They include the inconsistency of the surface and upper air observation networks, the nonuniform density of the basic observing network, and the difficulty of producing a reliable analysis when data from one or more key stations are missing.

CONTENTS

ABSTRACT iii

FIGURES vi

ABBREVIATIONS AND SYMBOLS viii

1. Introduction 1

2. Conclusions 2

3. Recommendations 3

4. The Interpolation Problem 4

Survey of possible techniques 4

Application of scan weighting to St. Louis 7

5. The Objective Analysis Program 9

Outline of the program 9

The interpolation function 12

The weighting function 14

The scan circle 16

The first guess field 28

Error rejection 31

6. Evaluation of the Program 34

REFERENCES 38

APPENDICES

A. Program Listings 39

B. Program Documentation 43

FIGURES

Number		Page
1.	Unequally spaced sampling stations (*) on an equally spaced grid	5
2.	Scan circle centered at a grid point, enclosing 5 sampling stations (*)	6
3.	St. Louis stations on a square grid (grid interval = 5 km). Surface stations are 101 to 125, upper air stations are 141 to 144	8
4.	Flow chart illustrating the logical structure of the Objective Analysis Program	10
5.	Scan circle centered at a sampling station (*), enclosing 29 grid points	11
6.	Interpolation to a station, Z(m+p,n+q), within a grid. Nine-point interpolation uses all nine grid points, whereas four-point interpolation uses the four grid points surrounding the interpolated point (*)	14
7.	Various weighting functions tested for use in scan weighting	15
8.	Objectively analyzed u-component field after first iteration. Scan radius is 17 grid units	18
9.	Objectively analyzed u-component field after second iteration. Scan radius is 12 grid units	19
10.	Objectively analyzed u-component field after third iteration. Scan radius is 7 grid units	20
11.	Objectively analyzed u-component field after fourth iteration. Scan radius is 2 grid units	21
12.	Station network for Figures 8 to 11. Data used are 975 mb winds (m/sec) of 20 February 1964, 1130Z as reported in Scoggins and Smith (1973). (See Section 6)	22

FIGURES (Continued)

Number		Page
13.	Objectively analyzed u-component field after four iterations. Data are the same as in Figures 9 to 12, but the scan radii are 11, 8, 5, and 2 grid units.	24
14.	Objectively analyzed temperature field using scan radii of 13, 10, 7, 4, and 1 grid units. Data used are 1500 CDT mean surface temperatures (^{o}F) for August 1972 as shown in Huff (1973). (See Section 6)	25
15.	Objectively analyzed temperature field using scan radii of 20, 15, 7, 4, and 1 grid units. In addition, all observations are not used for each iteration. Data are the same as in Figure 14.	26
16.	RAMS stations on a 100 x 100 km grid. The number for each station is the iteration in which the station is first used	27
17.	Objectively analyzed temperature field. This is like Figure 15, except for a different first guess field	29
18.	First guess field for Figure 17	30
19.	V-component grid with two rejected (in rectangles) data values (1.2 and -.1). Data used are RAMS winds (m/sec) for Hour 11, Day 67, 1975	32
20.	Hand analyzed temperature field, No. 1. Data used are 0600 CDT mean surface temperatures (^{o}F) for August 1972 as shown in Huff (1973)	35
21.	Hand analyzed temperature field, No. 2. Data are the same as in Figure 20	35
22.	Objectively analyzed temperature field using scan radii of 11, 8, 5, and 1.5 grid units. Contours drawn by hand to fit the objectively analyzed gridded values (not shown). Data are the same as in Figure 20	36

ABBREVIATIONS AND SYMBOLS

ABBREVIATIONS

km	--	kilometer
mb	--	millibar
NMC	--	National Meteorological Center
NSSFC	--	National Severe Storms Forecast Center
RAMS	--	Regional Air Monitoring System
RAPS	--	Regional Air Pollution Study

SYMBOLS

A_j	Value interpolated at data point j from surrounding grid values.
C_j	The difference between observed and interpolated values at data point j.
D_j	Observed value at data point j.
i	Index referring to grid point.
j	Index referring to data point.
J	The number of data points.
m	Integer values of horizontal coordinate on analysis grid.
MS	Mean square value of C_j.
n	Integer values of vertical coordinate on analysis grid.
p	Fractional distance so that m + p defines horizontal coordinate of data point j; see Figure 6.
q	Fractional distance so that n + q define vertical coordinate of data point j; see Figure 6.

ABBREVIATIONS AND SYMBOLS (Continued)

r_{ij}	Distance between grid point i and data point j.
R	Radius of the scan circle.
t	Tolerance criteria for rejecting observational errors.
u	West-east velocity component.
v	South-north velocity component.
X_i	Grid point value.
W_{ij}	The weighting function.
Z	A two-dimensional function.

SECTION 1

INTRODUCTION

The Regional Air Pollution Study (RAPS) was initiated in 1972 by the Environmental Protection Agency to collect the necessary data base for the development and verification of mathematical air quality models. The primary source of data for the RAPS program is a surface network of 25 automated meteorological and air quality monitoring stations located in St. Louis, MO. Data from the surface network are recorded routinely and checked for quality in St. Louis. The data to be used as input to the objective analysis program described here are hourly averages of wind and temperature from all stations in the network. The surface data are supplemented by a less dense network of upper air soundings. Routinely, these consist of four rawinsonde soundings per day at each of two locations (one urban and one rural), plus pibal wind soundings at these same two locations every hour that rawinsonde ascents are not scheduled. These two upper air sounding stations are supplemented during intensive periods of data collection by an additional two stations operating on the same schedule.

This report documents the development of an objective analysis program designed to interpolate meteorological variables to a regularly spaced grid from an irregularly spaced array of observations. Section 4 contains a discussion of the interpolation problem. The elements of the program are presented in Section 5 and an evaluation of the program is given in Section 6. The Appendices contain program listings and documentation.

SECTION 2

CONCLUSIONS

The program documented in this report, when properly applied to reliable data sets, should yield satisfactory analyses on the scale of RAPS. Error rejection can be accomplished whenever there are sufficient data to check on inter-station consistency. This can be done for the surface RAMS network but cannot be done with the sparse network utilized to collect the upper air data for RAPS. In the RAMS surface network, having reliable data at the outermost stations is crucial for a successful analysis of the entire field. This is due to the varying density of the observing network of the Regional Air Monitoring System.

SECTION 3

RECOMMENDATIONS

One of the special features of the Objective Analysis Program presented in this report is the ability to provide on a uniform grid an objective analysis from data recorded at a network of surface stations the density of which varies greatly over the analysis grid. This analysis generally provides gridded values which are consistent with the quality of the observed data. The variability of this analyzed field appears to reproduce the variability of the observed field over a range of scales. The procedure for accomplishing this is discussed in Section 5.

Unfortunately, an analysis of available surface data from the Regional Air Monitoring System (RAMS) reveals a considerable amount of missing data from one or more of the outlying stations. This is especially troublesome since the success of the present analysis technique is highly dependent on the presence of reliable data from the outermost ring of stations. It is strongly recommended, therefore, that throughout the remainder of RAPS special attention be given to maintaining the reliable operation of these key stations.

No matter how much care is taken to collect reliable data, periods will inevitably occur when data at key stations are missing. The objective analysis program presented here has no provision for missing data other than the trivial one of ignoring it. Ignoring missing data presents no problem in the center of the grid where stations are closely spaced. However, if the data observed at an outlying station is missing, the analysis may be seriously degraded. It is, therefore, recommended that some further attention be given to the impact of missing data on the quality of the analysis and that some way be found to provide to a potential user a measure of the reliability of the analysis.

SECTION 4

THE INTERPOLATION PROBLEM

SURVEY OF POSSIBLE TECHNIQUES

The numerical processing of meteorological information generally requires the data field to be defined on an equally spaced grid. However, meteorological observation stations are seldom spaced at regular intervals. Figure 1 shows an example of a grid to be defined from data recorded at several unequally spaced sampling stations. One effective technique is to subjectively analyze the field, drawing isolines based on the data at the stations. Grid point values can then be generated by manually interpolating between isolines. When large amounts of data are involved, as in RAPS, manual methods are not practical. An objective analysis procedure is necessary, and even then, the technique must be simple enough to run efficiently on a computer. Several possible objective analysis methods are discussed below:

Optimum Interpolation

This technique is attractive because the statistics of the naturally occurring variability of the parameter to be interpolated are used to define the interpolation function. Gandin (1965) discusses the overall optimum interpolation method, while Dartt (1972) gives an application to automated large-scale wind analysis. The overwhelming disadvantage of the technique is that the statistical properties (correlations and variances) necessary to specify the interpolation function are not known a priori as a function of such meteorological variables as lapse of temperature, wind speed, wind direction, etc. These statistical properties can only be deduced from a careful analysis of long records of data comparable to the data to be objectively analyzed.

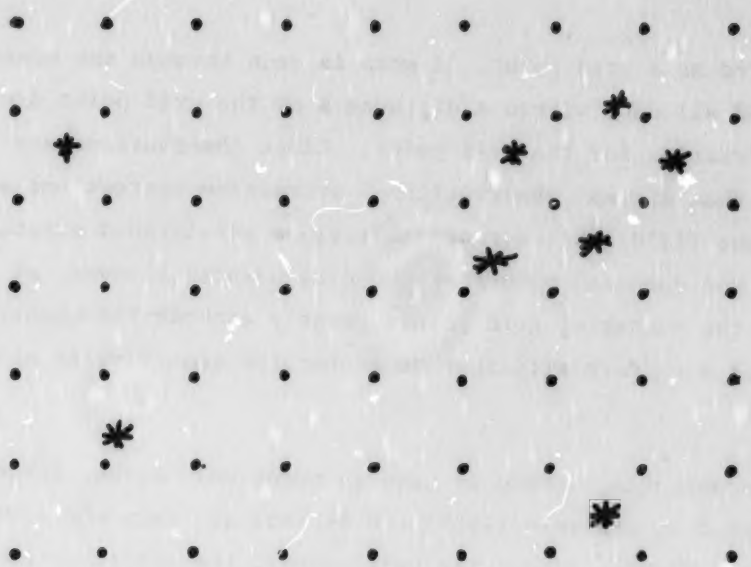

Figure 1. Unequally spaced sampling stations (✱) on an equally spaced grid.

Surface Fitting

Polynomials of a degree smaller than the number of data points can be computed and used to define meteorological parameters at grid points. Techniques vary from simple linear interpolation to complex multi-dimensional polynomials of higher degree. The polynomial approach can give exaggerated values at distances considerably removed from data points especially if these points are unevenly distributed over the grid. Also, if the distribution of data points varies considerably with time, there can be problems with time continuity of successive analyses. Computationally, polynomials of higher degree can be quite costly to evaluate.

Scan Weighting

This analysis method starts with a first guess field. In the usual application, each observation within a circle of influence provides a correction to the first guess grid point values. Figure 2 shows the scan

circle centered at a grid point. A scan is made through the station locations, and all data within a distance R of the grid point are used to compute a correction for the grid point. Close observations are weighted more heavily than distant observations. Successive corrections are made to the previous field until a reasonably close consistency exists between observations and computed values at the grid points (Belmont, et al., 1966). When the number of grid points greatly exceeds the number of observations, it is much more efficient to center the scan circles at observation points.

The scan weighting method is used in numerical weather forecasting and is described by Cressman (1959) and Bedient and Vederman (1964). The Joint Numerical Weather Prediction Unit adopted the method of Bergthorsson and Döös (1955) because the polynomial scheme previously in use ran into trouble in areas of sparse data. Bergthorsson and Döös (1955) point out that in such areas a previous analysis or even climatology may be a better representation of the field than an interpolated analysis. In the scan weighting scheme, the final analysis will strongly reflect the first guess field in areas with few data points.

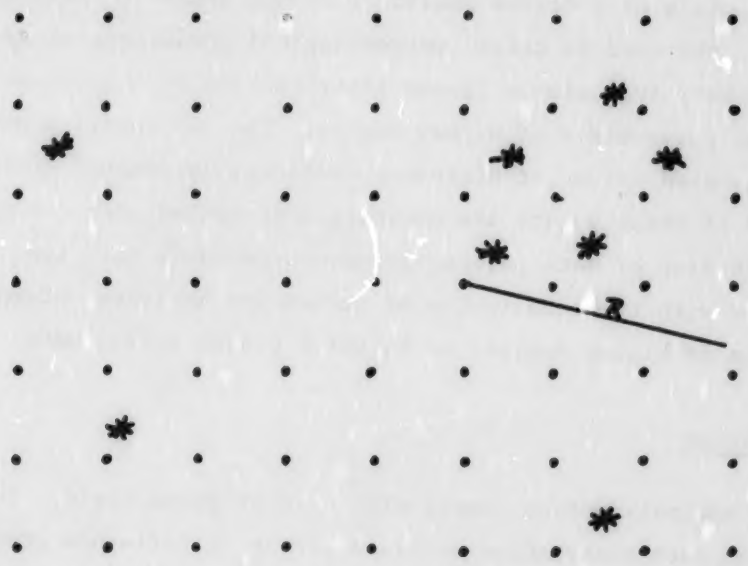

Figure 2. Scan circle centered at a grid point, enclosing 5 sampling stations (✱).

APPLICATION OF SCAN WEIGHTING TO ST. LOUIS

The scan weighting technique was chosen for the objective analysis of the St. Louis data because of its flexibility in accommodating various horizontal distributions of observations. Figure 3 shows the locations of the St. Louis stations on a 100 km square grid. The grid spacing is 5 km for a total of 21 times 21 = 441 grid points. The grid is positioned so that Station 101 is at the center grid point. The density of observational stations is greatest near the center of the grid and sparse at the external boundaries of the grid. Only 35% of the grid points are within the area enclosed by the stations. Data at the remaining 65% of the grid points are computed by extrapolation not interpolation. It is required that the horizontal scale of variability at grid points conforms to similar patterns indicated by the local distribution of observations. The small-scale variability that may occur near the middle of the grid should not propagate outwards to the external boundaries where there are few observations. This can be prevented by either of two methods: 1) using telescoping grids or 2) gradually adding observations at various iterations in the scan procedure.

In the case of telescoping grids, a complete large-scale analysis is made first on the largest mesh grid. Grid point values for the next smaller mesh grid are interpolated from this large-scale analysis. These interpolated values are then used as the first guess field for a complete re-analysis of observational data within the small-scale grid. This process can be repeated for as many nested grids as necessary.

A second approach is to use one small-scale grid and add observations at various iterations in the scan procedure. First, widely spaced observations and a large scan radius are used to approximate the field. This is successively followed by the addition of more observations and a reduction in scan radius until the field converges to the desired analysis. This procedure of simultaneously adding more data and shrinking the scan radius insures that the small-scale variability in areas of dense observations does not propagate into the surrounding area where there are few data. It is this approach which has been adopted here.

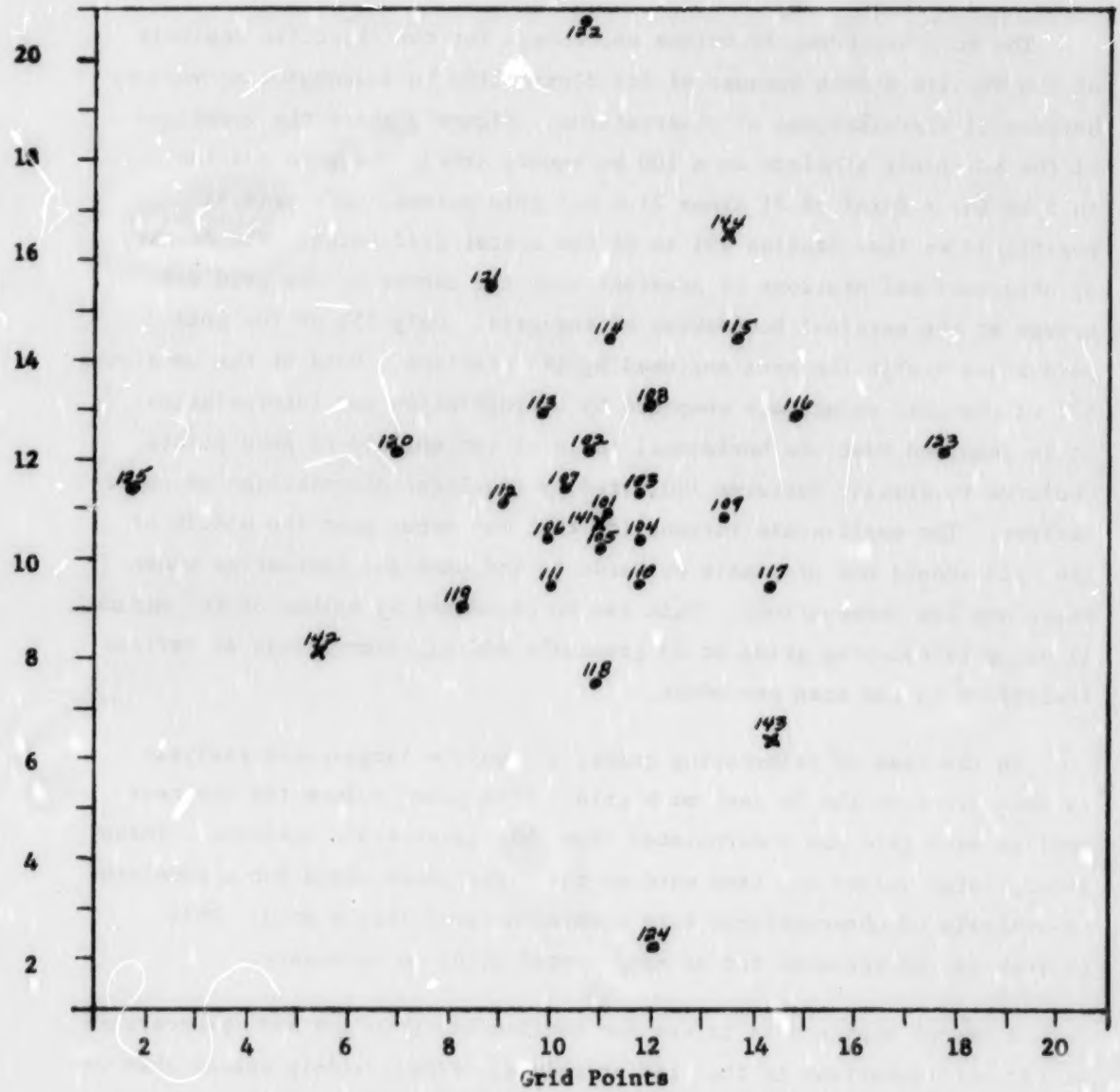

Figure 3. St. Louis stations on a square grid (grid interval = 5 km). Surface stations are 101 to 125, upper air stations are 141 to 144.

SECTION 5

THE OBJECTIVE ANALYSIS PROGRAM

OUTLINE OF THE PROGRAM

The objective analysis program, SUBROUTINE GRID, is listed in Appendix A. A flow chart outlining the logical construction of this routine is shown in Figure 4.

It is necessary for the objective analysis to start with a value at each grid point. This first guess field may be a previous analysis, a forecast, or simply a uniform field. Computations are initiated in the location of the first observational data point. A scan circle is centered at this data point as shown in Figure 5. The value A_j at the observation point is then interpolated from the initial guess field. If the observed value at the data point j is referred to as D_j, then the difference

$$C_j = D_j - A_j \tag{1}$$

provides a correction to be supplied to the grid point values, X_i, within a neighborhood of the data point. Note that i is an index associated with the grid points and j is an index associated with the observation points. The number of observation points is J.

The actual correction supplied to a grid point within the vicinity of data point j is weighted by the distance, r_{ij}, between the two points. This weighting function is given by

$$W_{ij} = \left[\frac{R^2 - r_{ij}^2}{R^2 + r_{ij}^2} \right]^4 \quad \text{provided} \quad r_{ij} \leq R, \tag{2}$$

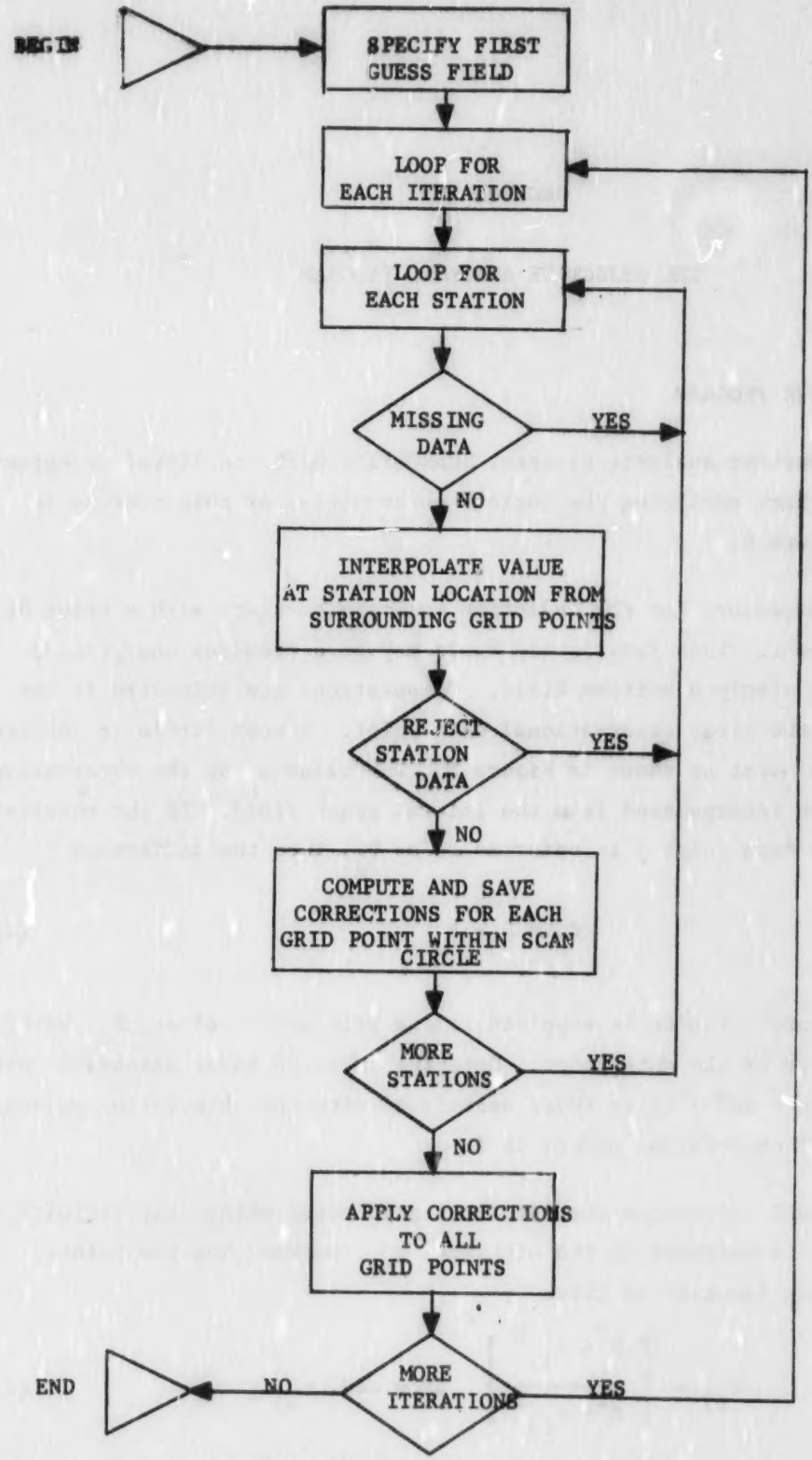

Figure 4. Flow chart illustrating the logical structure of the Objective Analysis Program.

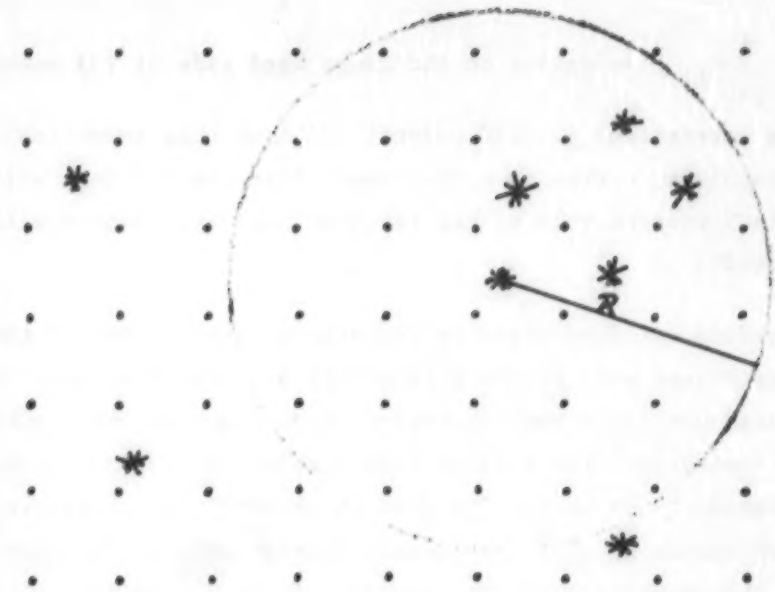

Figure 5. Scan circle centered at a sampling station (✱), enclosing 29 grid points.

where R is the preselected radius of the scan circle. If the distance r_{ij} exceeds R, then the weighting function is assigned a value of zero. Therefore, each grid point lying within a circle of radius R from data point j will receive a correction $W_{ij}C_j$. The objective analysis program computes corrections to all grid points lying within the scan circles surrounding each of the J data points. Only after corrections from all data points have been computed are the values of grid points adjusted. The new values, $X_{i_{new}}$ at each grid point are given by

$$X_{i_{new}} = X_{i_{old}} + \frac{\sum_{j=1}^{J} W_{ij}C_j}{\sum_{j=1}^{J} W_{ij}} . \qquad (3)$$

The quantity $\sum_{j=1}^{J} W_{ij}$ appearing on the right hand side of (3) serves to normalize the corrections to grid points. Without this normalization grid points receiving corrections from many close observations will be "over-corrected" whereas grid points far from any observations will be "under-corrected".

The procedure outlined above is repeated a fixed number of times or until the corrections made to the grid points are less than some small value. The analysis after each iteration is used as the first guess field for the next iteration. The size of scan circles is often decreased from one iteration to the next. The goal is to obtain an objective analysis of the gridded values which is reasonably consistent with the observed data field. The variability of the analyzed field should then be reasonably consistent with the variability of the observed data even when the density of the observing network varies over the grid.

THE INTERPOLATION FUNCTION

To establish the value A_j at each observation location (Equation 1), an interpolation is required from the surrounding grid points. Several options are available for defining this interpolation function which is used at each observation point before each scan iteration (Figure 4). Indeed, the National Meteorological Center (NMC) uses three different interpolation functions depending on the location of the observation on the analysis grid (McDonell, 1974). The three functions are a sixteen point 2-dimensional function, a four point 2-dimensional function and a three point arithmetic average.

Figure 6 illustrates the four point 2-dimensional (bilinear) interpolation function used in the analysis. It can be derived as follows. Let Z be a field defined on a square grid. The value $Z(m+p,n+q)$ is to be interpolated from four grid points. The values of p and q are measured as fractions of the grid length. One dimensional linear interpolation between point (m,n) and point (m+1,n) gives

$$Z(m+p,n) = Z(m,n) + p[Z(m+1,n) - Z(m,n)]. \quad (4)$$

Similarly, one-dimensional linear interpolation between point (m,n+1) and point (m+1,n+1) yields

$$Z(m+p,n+1) = Z(m,n+1) + p[Z(m+1,n+1) - Z(m,n+1)]. \quad (5)$$

Another linear interpolation between points (m+p,n) and (m+p,n+1) leads to

$$Z(m+p,n+q) = Z(m+p,n) + q[Z(m+p,n+1) - Z(m+p,n)]. \quad (6)$$

When (4) and (5) are substituted into (6), it is found that

$$Z(m+p,n+q) = [1-p][1-q]\, Z(m,n) + p[1-q]\, Z(m+1,n) + q[1-p]\, Z(m,n+1) + pq\, Z(m+1,n+1), \quad (7)$$

which is the desired relation. Note that

$$[1-p][1-q] + p[1-q] + q[1-p] + pq = 1. \quad (8)$$

Also, note that it is assumed here that all data points lie within the analysis grid. The objective analysis program has been tested using both four point and nine point interpolation functions. The four point function usually gives comparable results and is always faster.

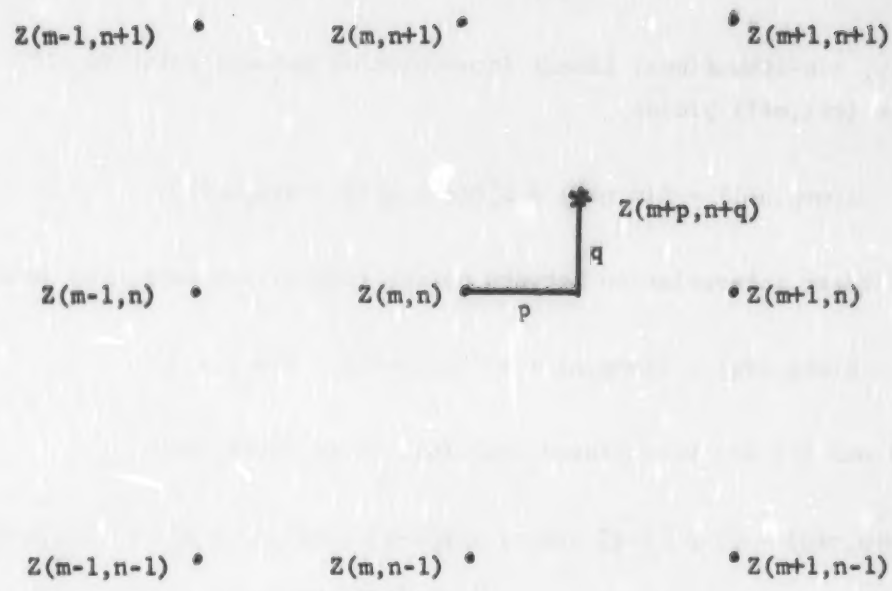

Figure 6. Interpolation to a station, Z(m+p,n+q), within a grid. Nine-point interpolation uses all nine grid points, whereas four-point interpolation uses the four grid points surrounding the interpolated point (*).

THE WEIGHTING FUNCTION

In order to obtain an analyzed field which is consistent with the field of observed data, it is necessary to weight the influence of a data point upon a given grid point according to the distance between the two points. Maximum weight at any grid point is given to the closest observing point. If no data point lies within a distance R of a given point, that initial grid point value will remain unchanged by the analysis.

The desired weighting function is a monotonically decreasing function of the distance r_{ij} between observation point j and grid point i. This function, W_{ij}, is defined to be one when r_{ij} is zero and vanishes when r_{ij} exceeds the scan-circle radius, R. Several alternative weighting functions are reproduced in Figure 7. They correspond to

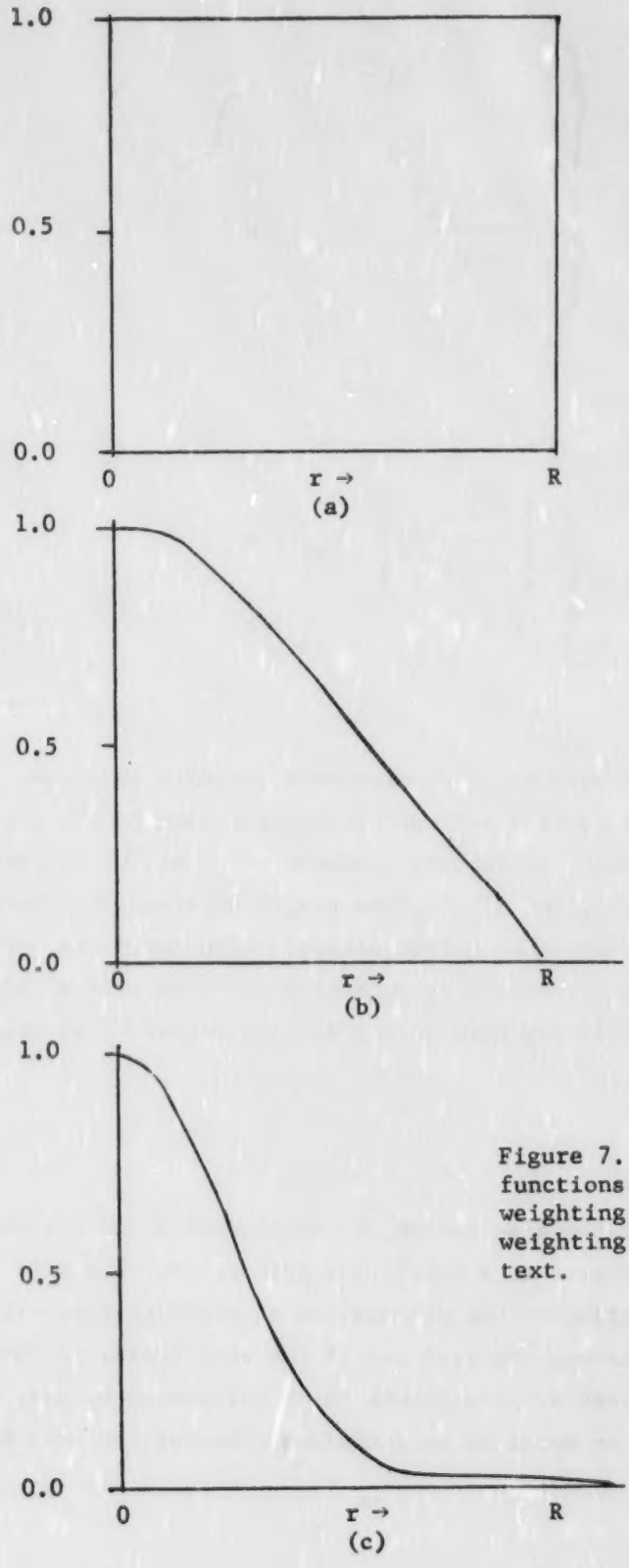

Figure 7. Various weighting functions tested for use in scan weighting. Equations defining weighting functions given in text.

$$W_{ij} = \begin{cases} 1 & r_{ij} \leq R \\ 0 & r_{ij} > R, \end{cases} \tag{9a}$$

$$W_{ij} = \begin{cases} \dfrac{R^2 - r_{ij}^2}{R^2 + r_{ij}^2} & r_{ij} \leq R \\ = 0 & r_{ij} > R \end{cases} \tag{9b}$$

and

$$W_{ij} = \begin{cases} \left[\dfrac{R^2 - r_{ij}^2}{R^2 + r_{ij}^2}\right]^4 & r_{ij} \leq R \\ = 0 & r_{ij} > R \end{cases} \tag{9c}$$

respectively. The weighting function used for this objective analysis program is defined by (9c) and reproduced in Figure 7. This function decreases rapidly with increasing distance which allows the use of a large scan radius, R, and still gives nearby observation points much more influence than distant observation points. When there are large areas of grid with few data points, it is necessary to begin with a large scan radius to ensure that all grid points are influenced by at least one observation point.

THE SCAN CIRCLE

The choice of the scan radius, R, determines which observation points enter into the analysis at a particular grid point. The size of the scan radius can even influence the propagation of observational errors. Random errors will be somewhat averaged out if the scan radius is chosen large enough for the values at grid points to be influenced by many observation points. However, an error in an individual observation will be confined

to only a few grid points if a small scan radius is used.

The best strategy for selecting the size of the scan radius and even varying it from one iteration to the next can be determined by experimenting with any given observation network. An especially attractive approach is to start with a fairly large scan radius for the first iteration and to reduce its size in successive iterations. This procedure has the effect of producing a smooth large-scale analysis while retaining the smaller scale variability in regions of dense observations. Figures 8 to 11 show an analysis after each of four iterations using scan radii of 17, 12, 7 and 2 grid units. The data are 975 mb winds on 20 February 1964, 1130Z, measured at the stations shown in Figure 12. Only the u-component analysis is given. The isolines of Figures 8 to 11 were drawn by manually interpolating (linearly) between the grid point values generated by the objective analysis. The observed data values are shown for reference.

The first guess field has value zero at each grid point. After one iteration (Figure 8) the basic pattern is defined, but the analysis does not match the observed data. With each succeeding iteration the analysis more closely matches the data. Since the field is quite smooth and the stations are fairly uniformly spaced, there is not much change from the third to the fourth iteration. A measure of the consistency of the analyzed field of grid values and the observed data field is given by MS, the mean square difference of the interpolated A_j and the observed D_j. That is, MS is defined by

$$MS = \frac{1}{J} \sum_{j=1}^{J} C_j^2 = \frac{1}{J} \sum_{j=1}^{J} (D_j - A_j)^2 . \qquad (10)$$

The convergence of the objective analysis can be evaluated by examining the computed MS values. In particular the analysis is said to converge if MS approaches zero in a finite number of iterations. In practice the objective analysis converges rapidly in a few iterations. For example, in the analyses corresponding to Figures 8 to 11, MS at the start of each of the four iterations is 32.12, 10.46, 1.62 and .01 respectively.

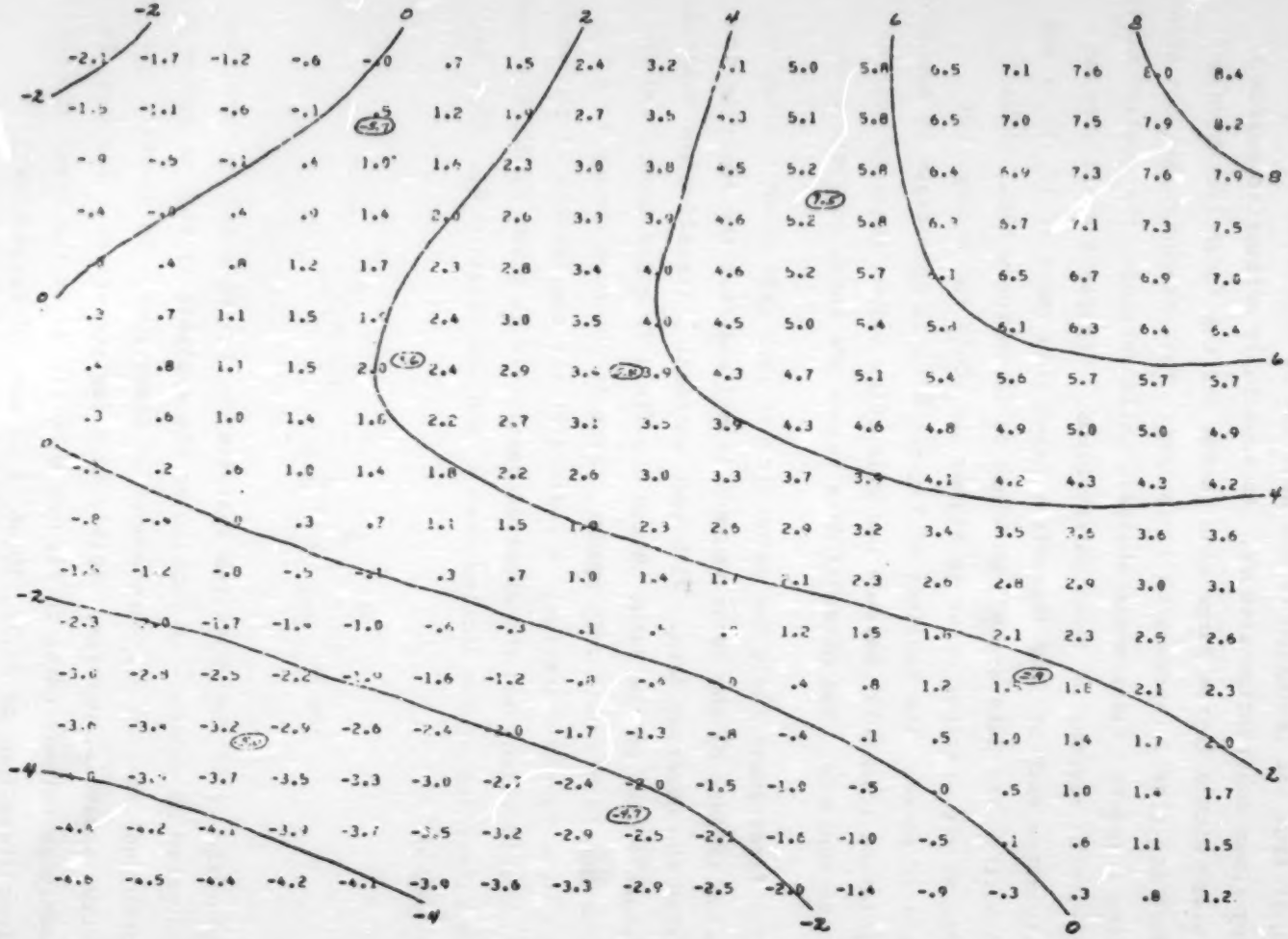

Figure 8. Objectively analyzed u-component field after first iteration. Scan radius is 17 grid units.

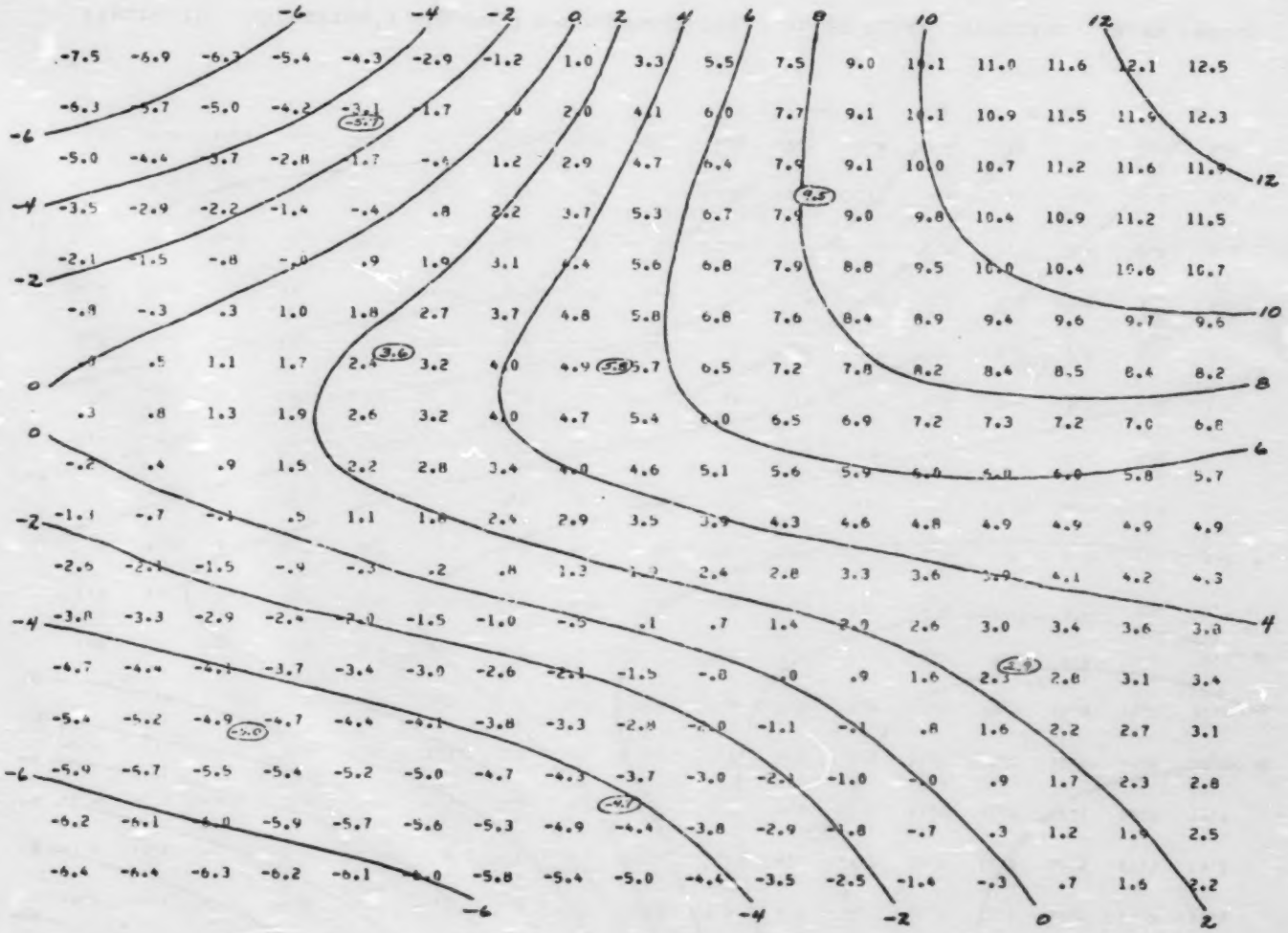

Figure 9. Objectively analyzed u-component field after second iteration. Scan radius is 12 grid units.

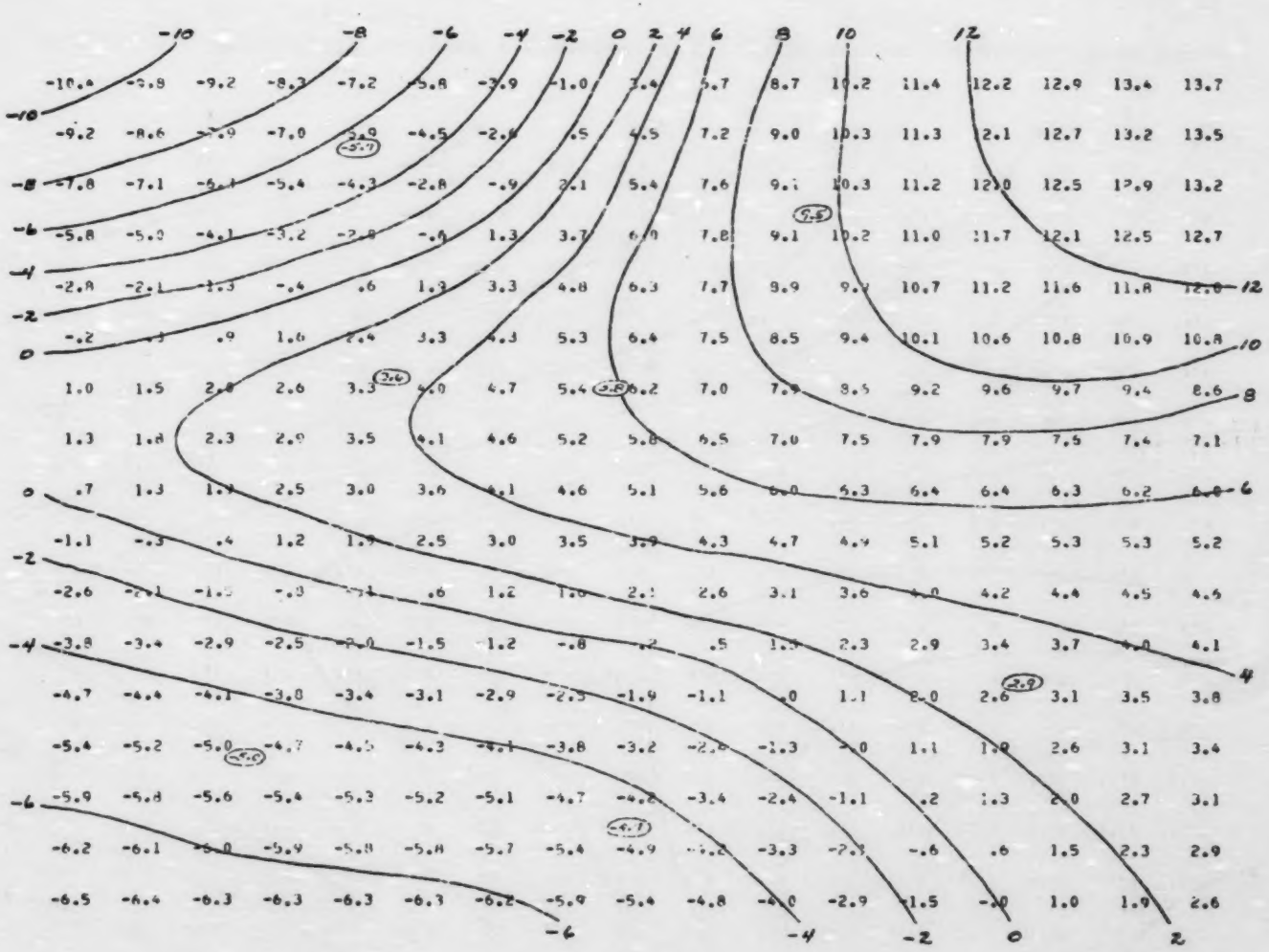

Figure 10. Objectively analyzed u-component field after third iteration. Scan radius is 7 grid units.

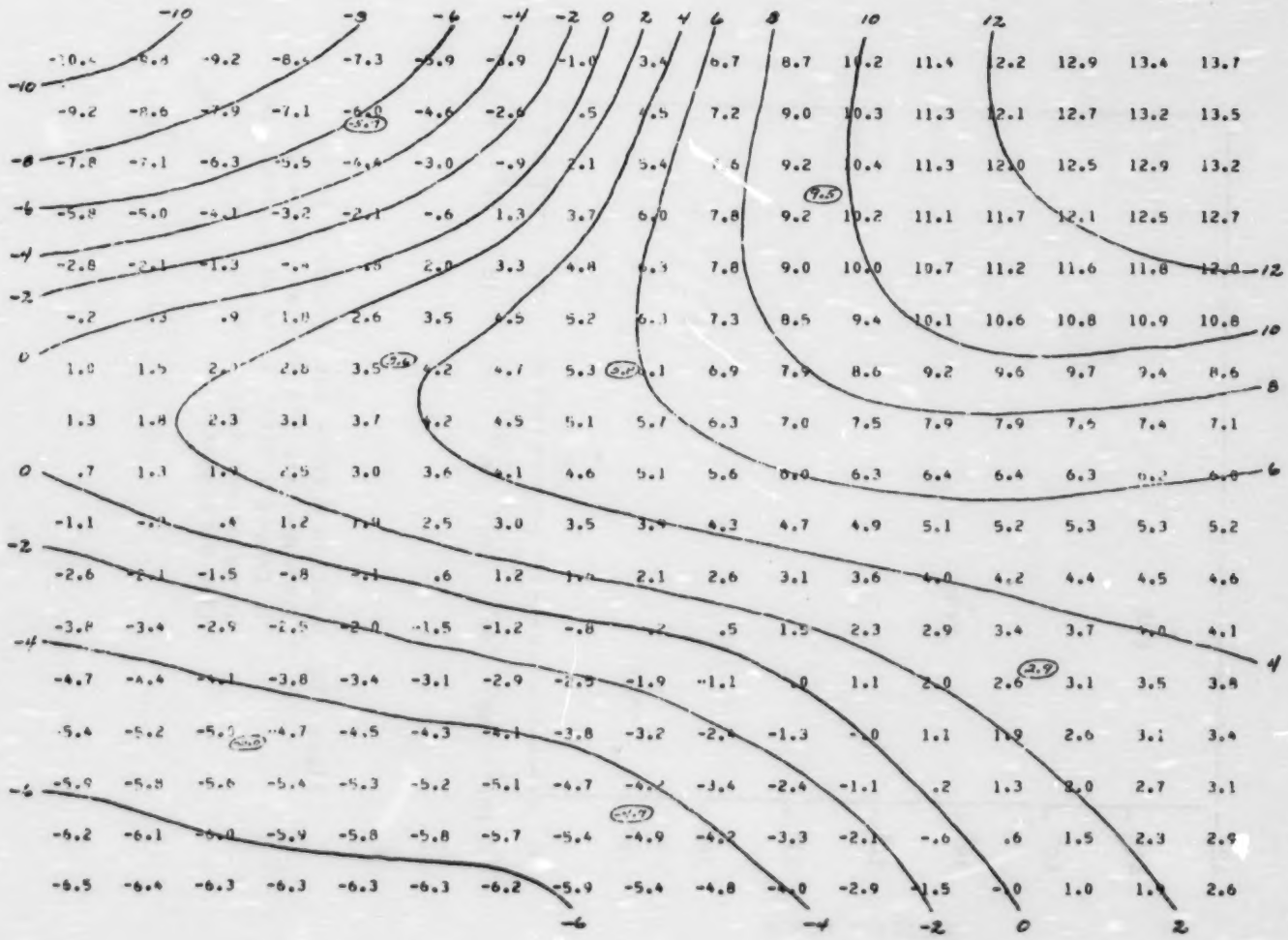

Figure 11. Objectively analyzed u-component field after fourth iteration. Scan radius is 2 grid units.

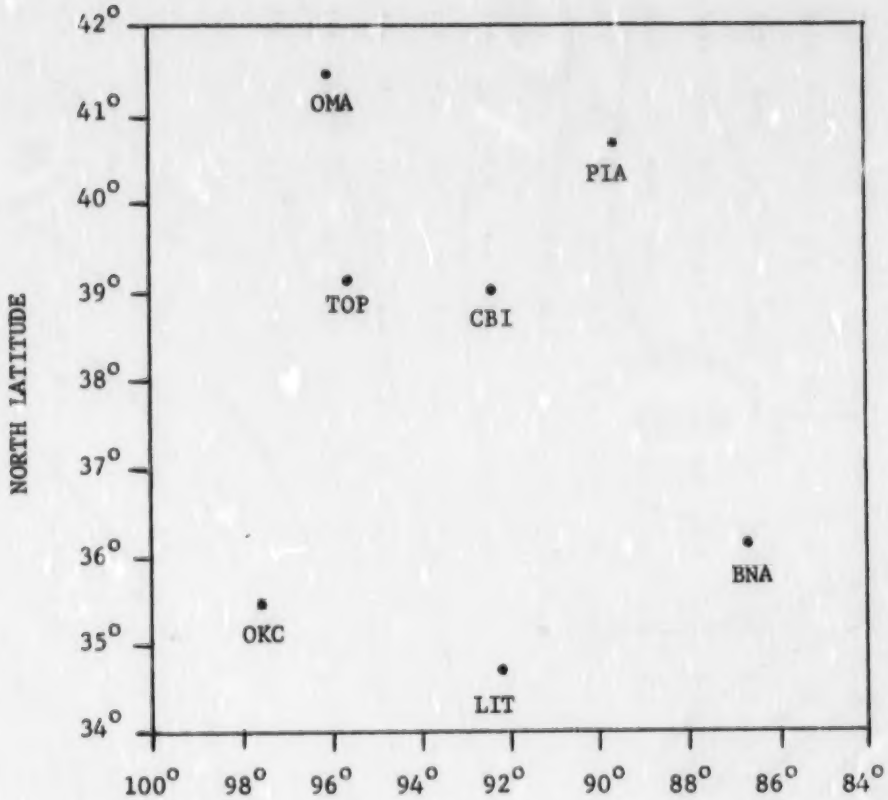

Figure 12. Station network for Figures 9 to 13. Data used are 975 mb winds (m/sec) of 20 February 1964, 1130Z as reported in Scoggins and Smith (1973). (See Section 6)

Convergence can be accelerated by using smaller scan sizes, but the analysis may be degraded. Figure 13 is the final analysis of the same data as in the preceding figure with R=11, 8, 5 and 2 grid units. In this case MS at the start of each iteration has values of 32.12, 1.82, .02 and .00 respectively. The convergence is faster but the analysis is somewhat different.

A special strategy has been worked out for applying the objective analysis algorithm to the observation network of the St. Louis Regional Air Monitoring System (RAMS), Figure 3. In the RAMS network the spacing between observing locations varies greatly over the grid. Most of the observations are concentrated near the center of the array. This large variation in density of the observational network poses special problems in the application of the objective analysis program. Figure 14 is an objective analysis of a temperature field over St. Louis using R=13, 10, 7, 4 and 1 grid units. The tight gradients in the northwest and southeast corners are generated by the program and are not seen in the data.

A much better objective analysis of the same data is shown in Figure 15. It was obtained after making two changes in the application of the program. For the first iteration only the four outermost observations are used. Twelve observations are used for iterations two and three. All twenty-five observations are used for iterations four and five. Figure 16 shows which stations are used during each iteration. In addition the scan radii for the first two iterations has been increased so that the entire sequence is R=20, 15, 7, 4, and 1 grid units.

The practice of mixing more data and simultaneously decreasing the scan radii in successive iterations appears to optimize the analysis of data observed from the non-uniformly spaced RAMS network. However, the application of this technique relies heavily upon the existence of reliable data at the outermost ring of stations. In this sense the outer stations become key stations in the analysis.

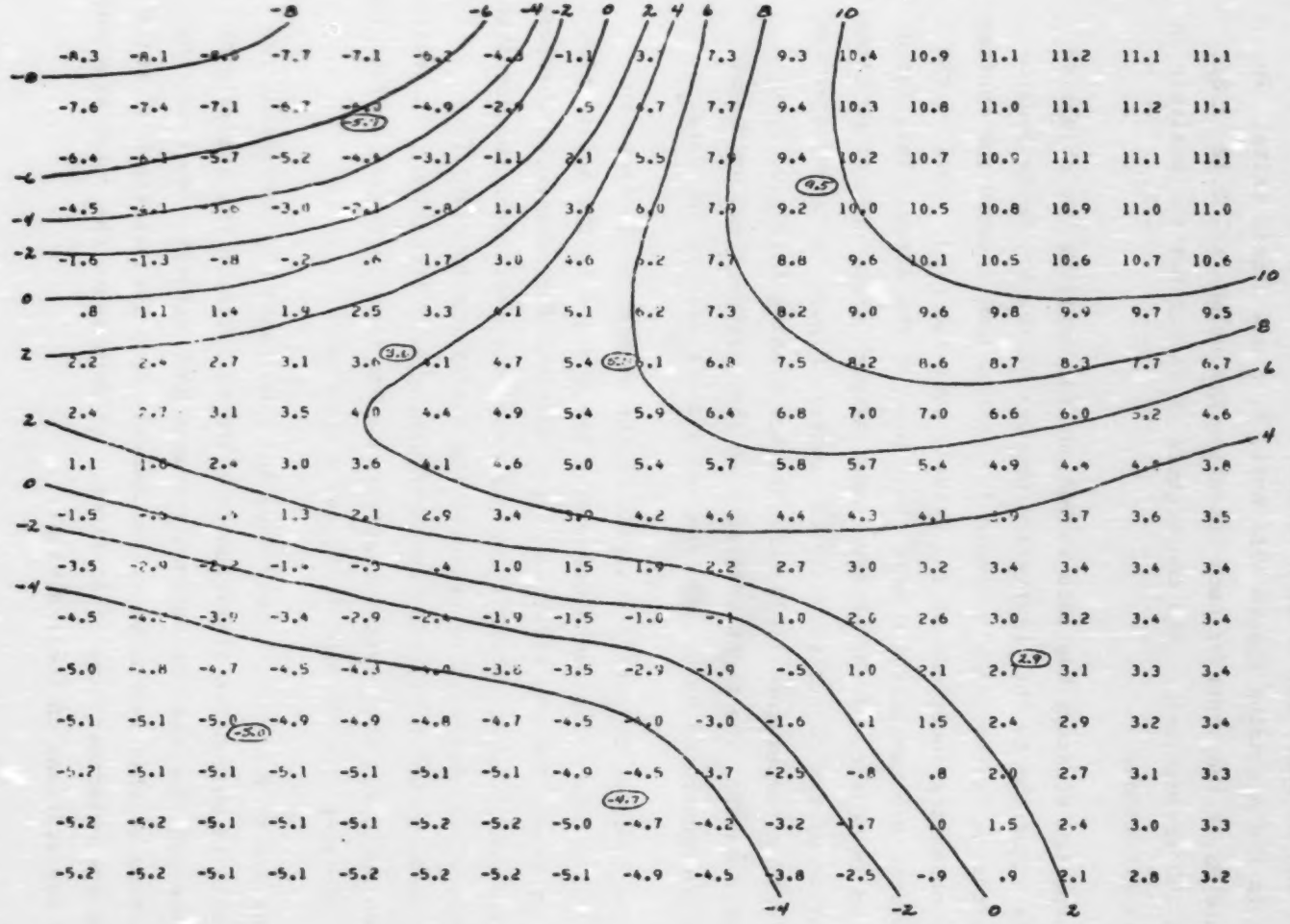

Figure 13. Objectively analyzed u-component field after four iterations. Data are the same as in Figures 9 to 12, but the scan radii are 11, 8, 5, and 2 grid units.

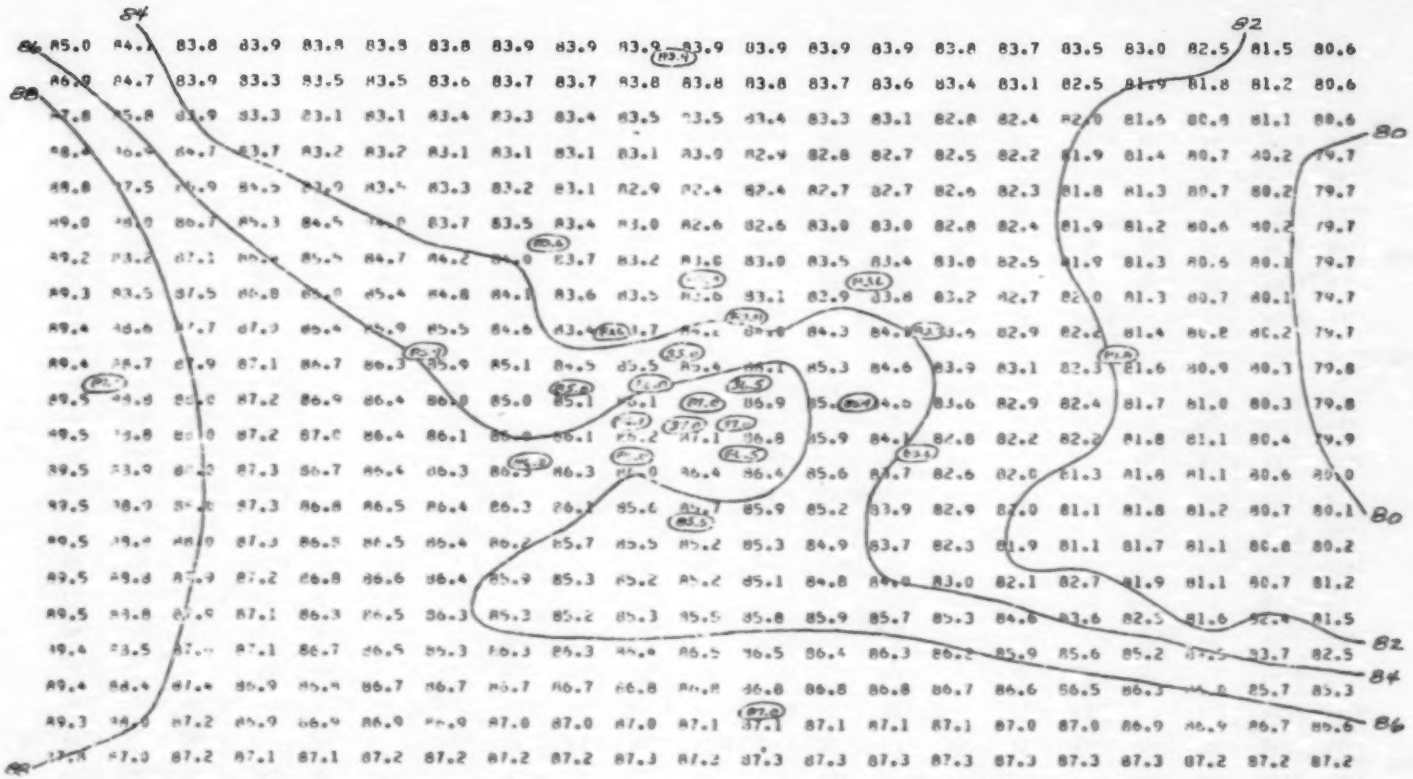

Figure 14. Objectively analyzed temperature field using scan radii of 13, 10, 7, 4, and 1 grid units. Data used are 1500 CDT mean surface temperatures (°F) for August 1972 as shown in Huff (1973). (See Section 6)

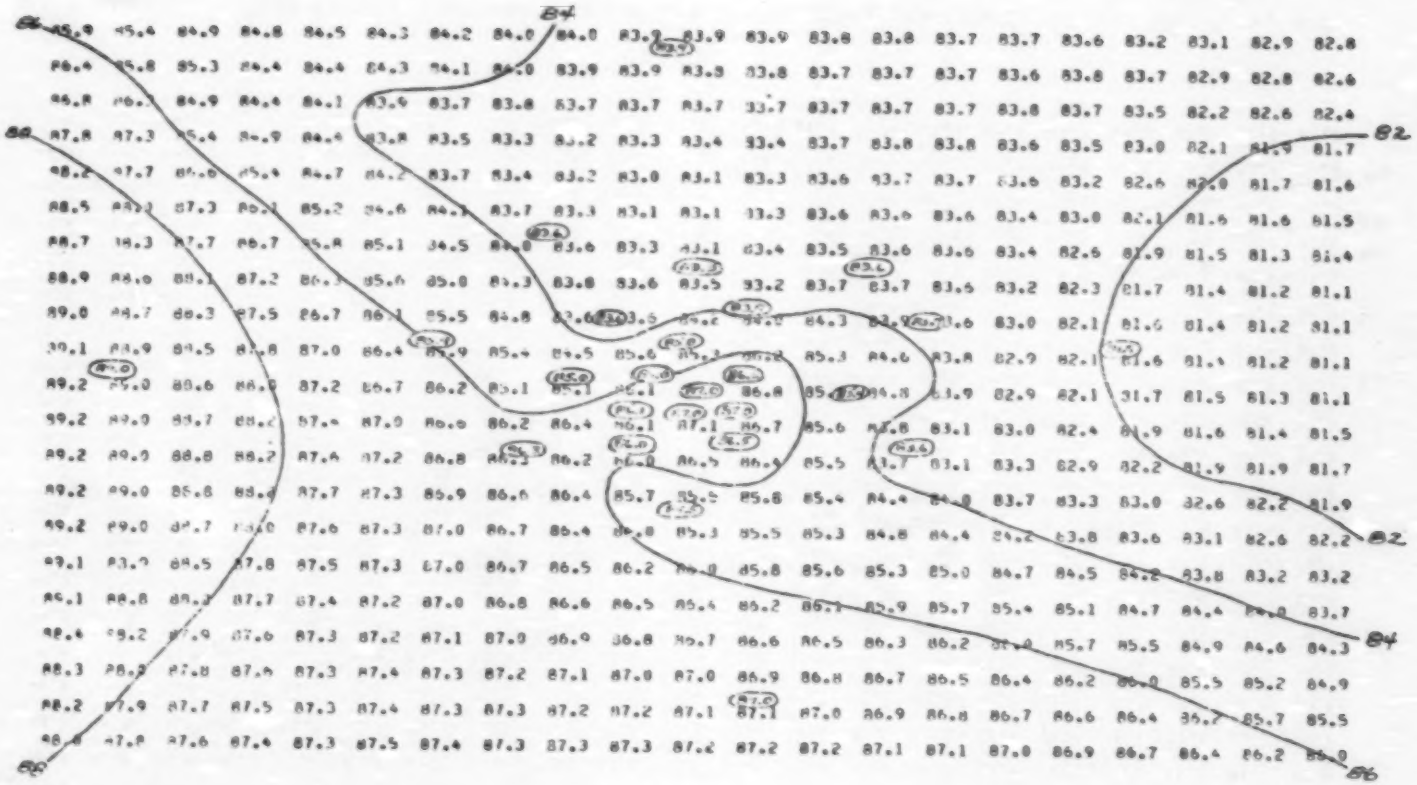

Figure 15. Objectively analyzed temperature field using scan radii of 20, 15, 7, 4, and 1 grid units. In addition, all stations are not used for each iteration. Data are the same as in Figure 14.

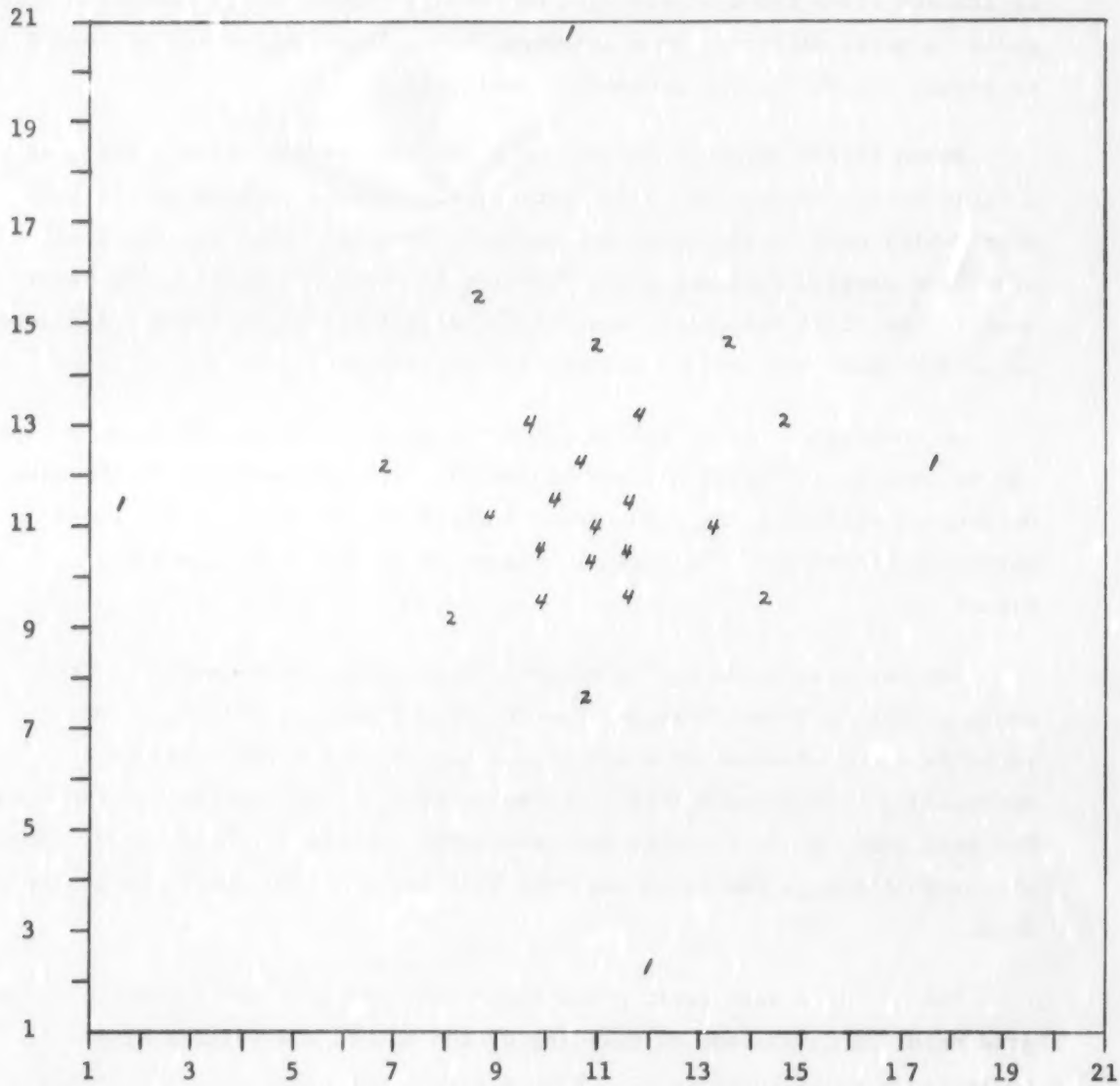

Figure 16. RAMS stations on a 100 x 100 km grid. The number for each station is the iteration in which the station is first used.

THE FIRST GUESS FIELD

The first guess field affects the convergence of the objective analysis procedure. When the first guess closely matches the data, MS (Equation 10) is small since the C_j are small. If all the C_j are zero, there is no need to iterate since the analysis will not change (Equation 3). When the first guess is quite different from the data, several iterations may be needed to reduce the MS to some acceptable small value.

Brown (1973) suggests the use of a forecast or persistence field as a first guess. Nearly any first guess field that is defined at all grid points will work in the objective analysis program. However, the field should be carefully chosen since features in the first guess field carry over to the final analysis. When tests using first guess fields of random noise are made, very noisy analyses are generated.

An example of the effect of the first guess field on the final analysis can be seen by comparing Figures 15 and 17 which are analyses of the same data using different first guesses. A field of all zeros is the first guess for Figure 15. The field in Figure 18 is the first guess for Figure 17.

According to Inman (1970) no good first guess fields exist for the National Severe Storms Forecast Center (NSSFC) use, so a field of all zeros is used. The use of a large scan size in the first iteration essentially defines each grid point value as the local average of the data. The zero first guess field is also preferred for the St. Louis grid. Then the variability in the final analysis will be only that specified in the data.

The use of a zero first guess field requires that the correction at a grid point be normalized by dividing by the sum of the weights of all the corrections contributed to it. With a zero first guess the C_j's in the first iteration are the data values. However, $\Sigma W_{ij}C_j$ could be a very large or a very small number. When the correction is normalized as in

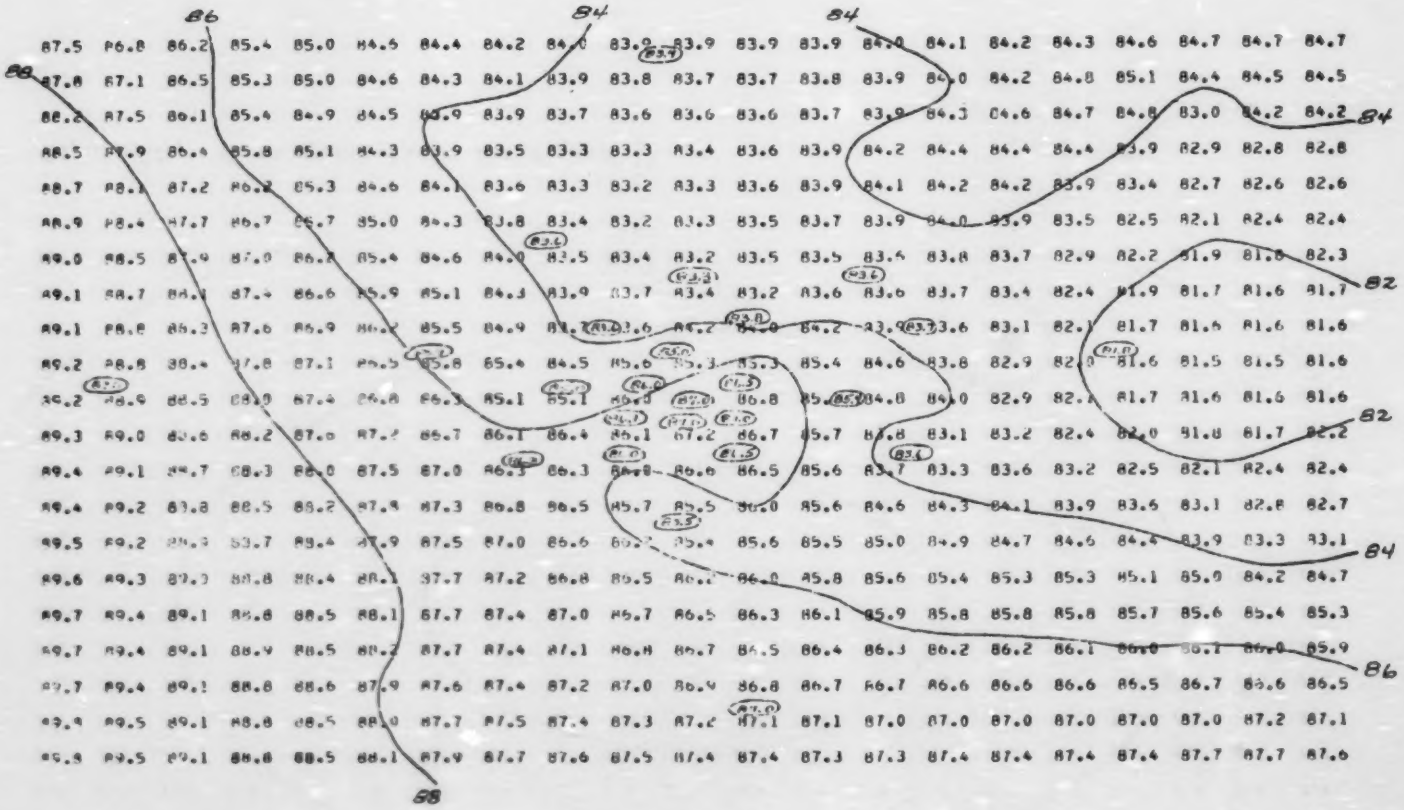

Figure 17. Objectively analyzed temperature field. This is like Figure 15, except for a different first guess field.

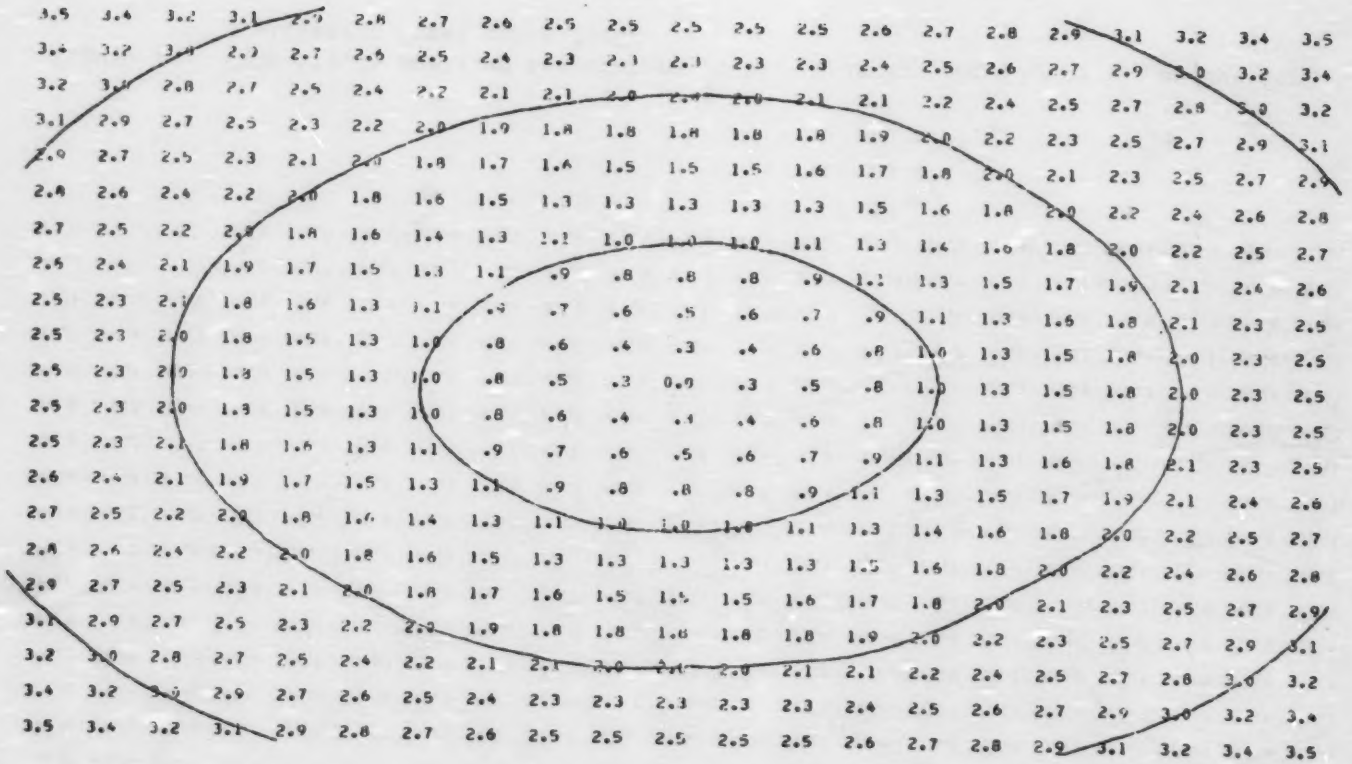

Figure 18. First guess field for Figure 17.

Equation (3) the new grid point values are of the proper magnitude.

ERROR REJECTION

Errors in data can sometimes be identified by examining C_j, the magnitude of the difference between the interpolated and actual values at data point j (see Equation 1). Since the grid point values are weighted averages of the data at nearby observation points, the station value, D_j, if in error, will be quite different than the value interpolated from the grid points, A_j. Therefore, if a suitable tolerance, t, can be found, an observation can be assumed to be in error (and ignored) whenever $C_j > t$. Cressman (1959) lists a set of liberal tolerances with t decreasing from one iteration to the next. In addition, an analyst monitoring the procedure can require the program to accept data tagged for rejection. Gandin (1965) prefers completely objective error checking since an analyst may accept or reject data simply because it does or does not fit his ideas.

The choice of t is obviously very important. When t is too large, very few errors are detected. When t is too small, good data are rejected. A value fixed for all cases is not desirable. A large value is needed when the field is highly variable and a smaller value is needed when the field is smooth. A tolerance based on the standard deviation, σ, of the observations seems reasonable. Tests using fictitious and real data, with and without known errors, show a value of $t = 2\sigma$ to be effective. Obvious errors are removed using this value. Brown (1973) gives throw criteria of 2.2σ where σ is computed for the C_j, not for the data. The throw criteria are restricted to predefined maxima since very bad observations generate an unrealistically large σ.

Figure 19 shows an objective analysis of the surface v-component of RAMS winds averaged over Hour 11, Day 67, 1975. Two of the 25 data values are obviously inconsistent. The -.1 value is from Station 109 where the wind speed was recorded as .1 mps for most of the day. The 1.2 value is from Station 115 where a wind direction of 102° was recorded while all other stations recorded directions near 340°. The program rejects the

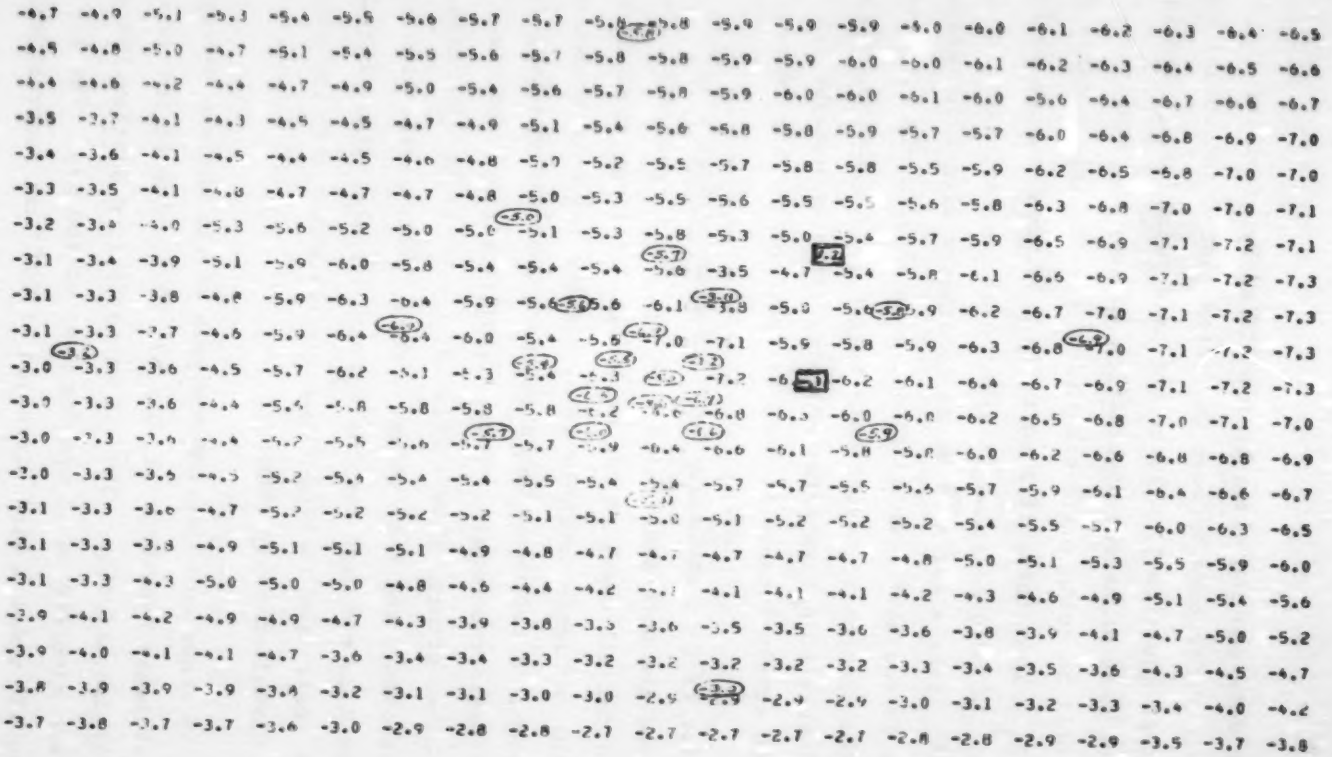

Figure 19. v-component grid with two rejected (in rectangles) data values (1.2 and -.1). Data used are RAMS winds (meters/sec) for Hour 11, Day 67, 1975.

two erroneous values and ignores them completely in arriving at the final analysis. The u-components for Station 109 and Station 115 must be rejected also since the two stations are identified as having erroneous winds for the observation time of Figure 19.

The standard deviation for Figure 19 is quite large because it is computed using the erroneous values. A preliminary screening may be used to remove obviously bad data before determining t. For this preliminary screening the mean and standard deviation σ are calculated for the set of all stations with data. Then all data more than 4σ from the mean are removed. The value 4σ is chosen because tests on St. Louis surface data show that most bad data are removed, while questionable data that may be good are left. Some good data are removed if 3σ or 3.5σ are used.

This error rejection scheme is applicable in St. Louis only to the surface data. There are just a few widely spaced upper air stations. In the situation where there are two soundings that do not agree, it is very difficult to determine if one is good and the other bad.

SECTION 6

EVALUATION OF THE PROGRAM

It would have been highly desirable to have had data available from the St. Louis RAPS program as the entire objective analysis program was being developed. As this data did not become available until rather late in the development effort, it was necessary to find alternate sources of data to check out the program. To this end, surface data for St. Louis was readily available from the Metromex program (Huff, 1973). This source was chosen because it presents the type of data which was expected from the St. Louis RAMS network. Data values were read from the published analysis of the mean temperature field at 1500 CDT for August 1972. The twenty-five data points coincide with the RAMS station locations. This was used as the source of data for Figures 14, 15, and 17.

The analysis of the mean temperature field at 0600 CDT for August 1972 was used in another test. Data values were read at twenty-five points corresponding to the Metromex hydrothermograph sites. These sites are more evenly distributed over the grid than are the RAMS stations. Hand analyses were drawn for maps on which the twenty-five data values were plotted. Figure 20 and Figure 21 are the patterns drawn by two different analysts. Figure 22 is the objective analysis of the same data using four iterations with scan radii of 11, 8, 5, and 1.5 grid units. Contours are reproduced which are consistent with the objectively analyzed gridded values. The objective analysis compares very well with the conventional (subjective) analysis.

Upper air data was taken from the NASA AVE I experiment as reported by Scoggins and Smith (1973). This report provides a convenient source of high frequency (three hour interval) rawinsonde soundings. The data are observations from the NOAA Synoptic stations in the southeastern United States. This data was utilized in Figures 8 to 11, and Figure 13.

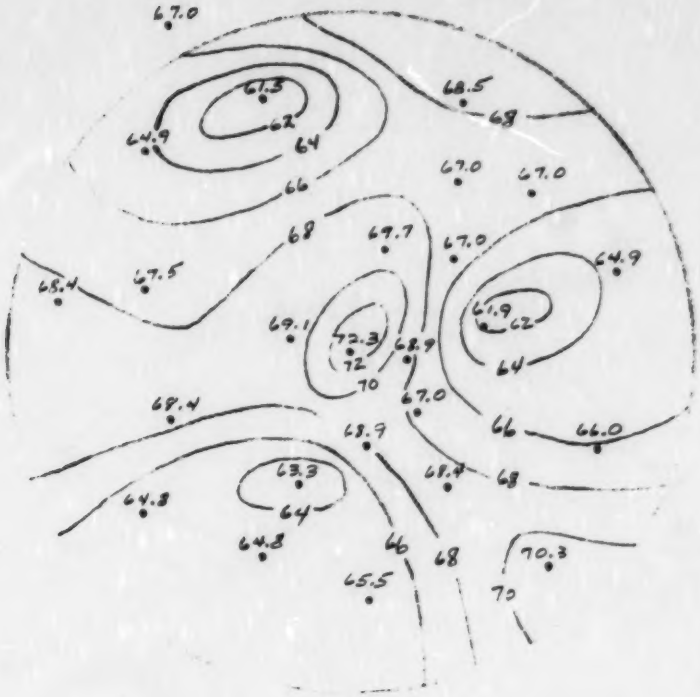

Figure 20. Hand analyzed temperature field, No. 1. Data used are 0600 CDT mean surface temperatures (^{o}F) for August 1972 as shown in Huff (1973).

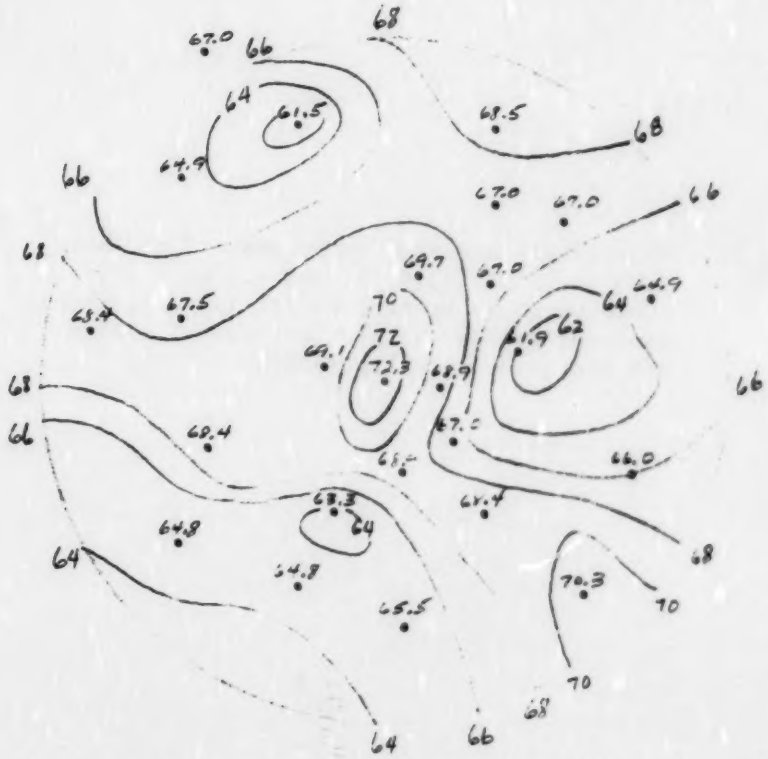

Figure 21. Hand analyzed temperature field, No. 2. Data are the same as in Figure 20.

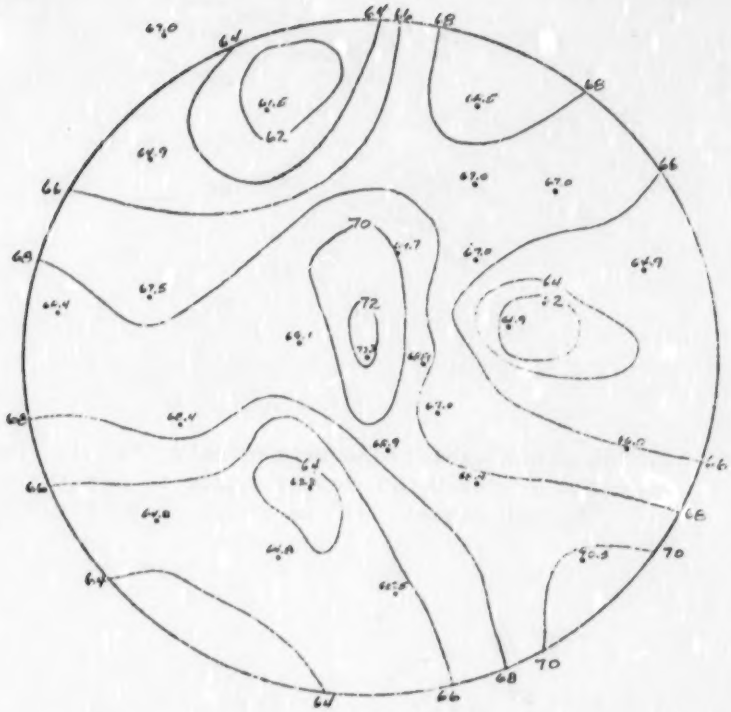

Figure 22. Objectively analyzed temperature field using scan radii of 11, 8, 5, and 1.5 grid units. Contours drawn by hand to fit the objectively analyzed gridded values (not shown). Data are the same as in Figure 20.

The Metromex and AVE data were used in developing and checking the objective analysis program because RAPS data were not available. The only RAPS data shown in this report are in Figure 19 which is an analysis of the v-components at the RAMS stations (Figure 3) averaged over Hour 11, Day 67, 1975. A full month of data, 14 July to 15 August, 1975, was made available in December, 1975. This period has some missing data. In addition, there are many bad values which will be removed by the error rejection procedure outlined above.

Unfortunately, the removal of bad data changes the observation network configuration. As mentioned in Section 5 the choice of scan sizes is dependent on the network configuration. Perhaps the scan radii presented in Section 5 will not produce a reliable analysis when many stations are missing. The first iteration, when only the outermost four stations are used, may be particularly troublesome. There are some observation times when three of the four outermost stations are missing.

REFERENCES

Bedient, H. A., and J. Vederman. Computer Analysis and Forecasting in the Tropics. Mon. Wea. Rev., 92, 565-577, 1964.

Belmont, A. D., L. W. Engberg, and W. C. C. Shen. Network Influence on Wind Interpolation in the Tropics. Final Report, Contract CWB-11164, Control Data Corp., Minneapolis, MN, 1966. 33 pp.

Bergthorsson, P., and B. R. Döös. Numerical Weather Map Analysis. Tellus, 7, 329-340, 1955.

Brown, J. F. Tropical Analysis Model. AFGWC Technical Memorandum 73-3, 1973. 20 pp.

Cressman, G. P. An Operational Objective Analysis System. Mon. Wea. Rev., 87, 367-374, 1959.

Dartt, D. G. Automated Streamline Analysis Utilizing "Optimum Interpolation." J. Appl. Meteor., 11, 901-908, 1972.

Gandin, L. S. Objective Analysis of Meteorological Fields. Israel Program for Scientific Translation, Jerusalem, 1965. 242 pp.

Huff, F. A., Ed. Summary Report of Metromex Studies, 1971-1972. Illinois State Water Survey, Urbana, 1973. 173 pp.

Inman, R. L. Papers on Operational Objective Analysis Schemes at the National Severe Storms Forecast Center. NOAA Technical Memorandum ERLTM-NSSL 51, 1970. 97 pp.

McDonell, J. E. Notes on Operational Objective Analysis Procedures. Prepared for "Advanced Prediction Techniques Course" at National Weather Service Headquarters, 1974. 76 pp.

Scoggins, J. R., and O. E. Smith. Data for First NASA Atmospheric Variability Experiment (AVE I) Part I: Data Tabulation. NASA Technical Memorandum X-2938, 1973. 682 pp.

```
      SUBROUTINE GRID(NDATA,XCOORD,YCOORD,STNVAL,
     1                IDIM,JDIM,GRDVAL,SUMWT,SUMCOR,
     2                ISTART,IEND,JSTART,JEND,
     3                NSCANS,RSCAN,CRIT,REJECT)
C
C     *GRID WILL TRANSFORM UNEQUALLY SPACED STATION DATA TO EQUALLY
C      SPACED GRID POINTS.  WHEN GRID IS CALLED THERE MUST BE A
C      VALUE FOR EACH GRID POINT.  THE FIRST GUESS FIELD MAY BE A
C      PREVIOUS ANALYSIS, A FORECAST, OR SIMPLY A CONSTANT.  THE GRID
C      MUST BE ARRANGED SO THAT GRID POINTS SURROUND EACH DATA POINT.
C
C     *NDATA IS THE NUMBER OF STATIONS WITH DATA, XCOORD AND YCOORD THE
C      STATION COORDINATES IN GRID UNITS RELATIVE TO GRID POINT (1,1),
C      AND STNVAL THE DATA VALUES AT THE STATIONS.
      DIMENSION XCOORD(NDATA),YCOORD(NDATA),STNVAL(NDATA)
C
C     *IDIM AND JDIM ARE THE DIMENSIONS OF THE GRID POINT ARRAYS.  GRDVAL
C      HOLDS THE GRID POINT DATA.  SUMWT AND SUMCOR ARE ARRAYS OF THE
C      SAME DIMENSIONS THAT MUST BE PROVIDED FOR TEMPORARY STORAGE.
      DIMENSION GRDVAL(IDIM,JDIM),SUMWT(IDIM,JDIM),SUMCOR(IDIM,JDIM)
C
C     *ISTART, IEND, JSTART, JEND ARE THE RANGE OF SUBSCRIPTS FOR THE
C      PART OF ARRAYS GRDVAL, SUMWT, AND SUMCOR THAT ARE ACTUALLY USED.
C
C     *NSCANS IS THE MAXIMUM NUMBER OF ITERATIONS TO BE USED, RSCAN HOLDS
C      THE RADIUS FOR EACH SCAN CIRCLE (MEASURED IN GRID UNITS), AND
C      CRIT IS A CONVERGENCE CRITERION - SEE BELOW.
C      REJECT IS AN ERROR REJECTION CRITERION.
      DIMENSION RSCAN(NSCANS)
C
C     *NPTS AND ERROR ARE USED TO MEASURE CONVERGENCE
      DATA NPTS/0/,ERROR/0/

      DO 10 I=ISTART,IEND
      DO 10 J=JSTART,JEND
      SUMWT(I,J)=0
 10   SUMCOR(I,J)=0
C
C     *ITER COUNTS THE NUMBER OF ITERATIONS
      ITER=0
 11   ITER=ITER+1
      IF(ITER.GT.NSCANS)GO TO 99
C
C     *RSQ IS RADIUS SQUARED OF THE CURRENT SCAN CIRCLE.
      RSQ=RSCAN(ITER)*RSCAN(ITER)
      ERROR=0
      NPTS=0
C
C     *LOOP FOR EACH DATA POINT.  999.0 REPRESENTS MISSING.
      DO 50 NDT=1,NDATA
      IF(STNVAL(NDT).EQ.999.0)GO TO 50
C
C     *FIND INDEXES OF NEAREST GRID POINT IN DIRECTION OF (1,1)
      I=XCOORD(NDT)+1.
      J=YCOORD(NDT)+1.
```

```
C
C     *FIND X AND Y DISTANCE FROM GRID POINT (I,J)
      P=XCOORD(NDT)-(I-1)
      Q=YCOORD(NDT)-(J-1)
C
C     *USE FOUR POINT BILINEAR FORMULA TO INTERPOLATE A VALUE AT
C      THE STATION LOCATION FROM FOUR SURROUNDING GRID POINTS.
C      IF A VALUE CANNOT BE INTERPOLATED, THE STATION IS SKIPPED.
      IF(I.LT.ISTART)GO TO 50
      IF(I+1.GT.IEND)GO TO 50
      IF(J.LT.JSTART)GO TO 50
      IF(J+1.GT.JEND)GO TO 50
      STNINT=GRDVAL(I,J)*(1.-P)*(1.-Q)
     1       +GRDVAL(I+1,J)*P*(1.-Q)
     2       +GRDVAL(I,J+1)*Q*(1.-P)
     3       +GRDVAL(I+1,J+1)*P*Q
C
C     *FIND DIFFERENCE BETWEEN INTERPOLATED AND OBSERVED VALUE AT
C      AT THE STATION.  IF THE DIFFERENCE IS LARGER THAN REJECT,
C      THE OBSERVED VALUE IS ASSUMED TO BE IN ERROR.
      DIFF=STNVAL(NDT)-STNINT
      IF(ABS(DIFF).GT.REJECT)GO TO 50
C
C     *THE MEAN SQUARE DIFFERENCE IS USED BELOW
      ERROR=DIFF*DIFF+ERROR
      NPTS=NPTS+1
C
C     *A CORRECTION IS NOW COMPUTED FOR EACH GRID POINT WITHIN A CIRCLE
C      OF RADIUS RSCAN(ITER) CENTERED AT THE STATION.  GENERATE A SCAN
C      SQUARE ENCLOSING THE SCAN CIRCLE.
      INDEX1=XCOORD(NDT)+1.-RSCAN(ITER)
      IF(INDEX1.LT.ISTART)INDEX1=ISTART
      INDEX2=XCOORD(NDT)+1.+RSCAN(ITER)
      IF(INDEX2.GT.IEND)INDEX2=IEND
      INDEX3=YCOORD(NDT)+1.-RSCAN(ITER)
      IF(INDEX3.LT.JSTART)INDEX3=JSTART
      INDEX4=YCOORD(NDT)+1.+RSCAN(ITER)
      IF(INDEX4.GT.JEND)INDEX4=JEND

      DO 40 M=INDEX1,INDEX2
      DO 30 N=INDEX3,INDEX4
C
C     *FIND DISTANCE SQUARED FROM STATION TO GRID POINT (M,N).
      DX=XCOORD(NDT)-(M-1)
      DY=YCOORD(NDT)-(N-1)
      DSQ=DX*DX+DY*DY
C
C     *GENERATE WEIGHT.  IF XNUM IS NEGATIVE, GRID
C      POINT (M,N) IS OUTSIDE THE SCAN CIRCLE.
      XNUM=RSQ-DSQ
      IF(XNUM.LT.0)GO TO 30
      XDEN=RSQ+DSQ
      WT=XNUM/XDEN
      WT=WT*WT
      WT=WT*WT
```

```
C
C     *CORRECTIONS ARE NOT APPLIED NOW SINCE A GIVEN
C      GRID POINT MAY GET CORRECTIONS FROM SEVERAL DATA POINTS.
       SUMWT(M,N)=SUMWT(M,N)+WT
       SUMCOR(M,N)=SUMCOR(M,N)+WT*DIFF
 30    CONTINUE
 40    CONTINUE
C
C     *END OF LOOP THROUGH DATA POINTS.
 50    CONTINUE
C
C     *NOW APPLY CORRECTIONS TO GRID POINTS
       DO 80 M=ISTART,IEND
       DO 70 N=JSTART,JEND
       IF(SUMWT(M,N).EQ.0)GO TO 70
       GRDVAL(M,N)=GRDVAL(M,N)+SUMCOR(M,N)/SUMWT(M,N)
       SUMWT(M,N)=0
       SUMCOR(M,N)=0
 70    CONTINUE
 80    CONTINUE
C
C     *IF THE MEAN SQUARE DIFFERENCE IS SMALL ENOUGH, STOP ITERATING.
       IF(NPTS.GT.0)ERROR=ERROR/NPTS
       IF(ERROR.GT.CRIT)GO TO 11

 99    RETURN
       END
```

```
      FUNCTION TRILI(A,L,M,N,X,Y,Z)
      DIMENSION A(L,M,N)
C
C     A IS AN ARRAY OF DIMENSION (L,M,N) HOLDING 3-DIMENSIONAL GRID
C     POINT DATA.  TRILI WILL INTERPOLATE A VALUE AT THE INTERNAL
C     POINT (X,Y,Z) WHERE X, Y, AND Z ARE MEASURED RELATIVE TO (1,1,1).
C     TRILI USES TRILINEAR INTERPOLATION.  IT REDUCES TO
C     2-DIMENSIONAL BILINEAR INTERPOLATION WHENEVER Z IS AN INTEGER.
C
C     FIND INDEXES OF NEAREST GRID POINT IN THE DIRECTION OF (1,1,1)
C     AND THE P, Q AND R DISTANCES FROM THAT GRID POINT.
      I=X+1.
      P=X-(I-1)
      J=Y+1.
      Q=Y-(J-1)
      K=Z+1.
      R=Z-(K-1)

      TRILI=(1.-R)*(P*(Q*A(I+1,J+1,K)+(1.-Q)*A(I+1,J,K))
     1             +(1.-P)*(Q*A(I,J+1,K)+(1.-Q)*A(I,J,K)))
C
C     R IS 0 IF K IS AN INTEGER
      IF(R.EQ.0)RETURN

      TRILI=TRILI+R*(P*(Q*A(I+1,J+1,K+1)+(1.-Q)*A(I+1,J,K+1))
     1              +(1.-P)*(Q*A(I,J+1,K+1)+(1.-Q)*A(I,J,K+1)))

      RETURN
      END
```

APPENDIX B: PROGRAM DOCUMENTATION

The objective analysis program, SUBROUTINE GRID, is listed in Appendix A. GRID was written for the CDC 6600. However, care was taken to use FORTRAN statements that are also valid on the Univac 1110. A useful reference is Garner (1973) which lists CDC and Univac FORTRAN differences. A test run has been made on the EPA Univac 1110, and SUBROUTINE GRID compiled and executed correctly.

The following elements are parameters of SUBROUTINE GRID and must be supplied by the calling routine:

NDATA	-	The number of data stations
XCOORD, YCOORD	-	Arrays of station coordinates measured relative to grid point (1,1). e.g., grid point (5,7) has XCOORD = 4 and YCOORD = 6.
STNVAL	-	Array of data values. 999.0 = missing.
IDIM, JDIM	-	The dimensions of arrays GRDVAL, SUMWT, and SUMCOR
GRDVAL	-	Grid point array. On entry to GRID, GRDVAL must contain the first guess field (see Section III-E). On exit from GRID, GRDVAL contains the analysis.
SUMWT, SUMCOR	-	Scratch arrays provided by the calling routine. The arrays need not be defined before GRID is called.
ISTART, IEND, JSTART, JEND	-	The range of subscripts for the parts of arrays GRDVAL, SUMWT, and SUMCOR that are actually used. $1 \leq ISTART \leq IEND \leq IDIM$ $1 \leq JSTART \leq JEND \leq JDIM$. Usually ISTART = JSTART = 1, IEND = IDIM, and JEND = JDIM.

NSCANS - The number of iterations.

RSCAN - Array of scan radii measured in grid units (see Section III-D).

CRIT - The convergence criterion (see Section III-D). GRID will stop iterating when MS (see Equation (5)), the mean square difference at the stations, is less than CRIT. Usually CRIT = 0 so that all NSCANS iterations are completed.

REJECT - Reject value (see Section III-F). If the first guess field is zero, any data value greater than REJECT will be rejected.

The following elements are programmed into SUBROUTINE GRID:

- The interpolation function (see Section III-B).
- The weighting function (see Section III-C).

A sample run using SUBROUTINE GRID is given in Appendix C. The execution time is proportional to the number of stations and the number of grid points and the number of iterations. GRID uses about .56 seconds of CDC 6600 central processor time for this sample run of 25 stations, 529 grid points, and 5 iterations.

FUNCTION TRILI is also listed in Appendix A. TRILI was developed along with SUBROUTINE GRID, but it is not actually used here. TRILI may be used for bilinear interpolation or for the extension to three dimensions, trilinear interpolation. (See Figure 6 for the derivation of the bilinear interpolation formula.)